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THE RELATIONS BETWEEN SOFT L-FUZZY TOPOGENOUS ORDERS AND SOFT L-FUZZY PRE-UNIFORMITIES

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Abstract: In this paper, we investigate the relations between soft L-fuzzy topogenous orders and soft L-fuzzy pre-uniform spaces. We give their examples.

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1. Introduction

Recently, Molodtsov [12] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [1,3]. Pawlak's rough set [13,14] can be viewed as a special case of soft rough sets [3]. The topological structures of soft sets have been developed by many researchers [2,7-10,15-18].

On the other hand, Hájek [4] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structures [5,7-10]. Kim [8] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice. Kim [7-10] introduced the soft topological structures, L-fuzzy quasi-uniformities and soft L-fuzzy topogenous orders in complete residuated lattices.

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2. Preliminaries

Definition 1. [4,5] An algebra $(L, \wedge, \vee, \odot, \rightarrow, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L = (L, \leq, \vee, \wedge, 1, 0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \rightarrow z$ for $x, y, z \in L$.

In this paper, we assume that $(L, \leq, \odot, \rightarrow, \oplus, ^*)$ is a complete residuated lattice with an order reversing involution * which is defined by $x \oplus y = (x^* \odot y^*)^*$ and $x^* = x \rightarrow 0$.

Lemma 2. [4,5] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties:

(1) $1 \rightarrow x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \rightarrow y \leq x \rightarrow z$ and $z \rightarrow x \leq y \rightarrow x$,

(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(6) $x \oplus (\bigwedge_i y_i) = \bigwedge_i (x \oplus y_i)$,

(7) $x \rightarrow (\bigwedge_i y_i) = \bigwedge_i (x \rightarrow y_i)$,

(8) $(\bigvee_i x_i) \rightarrow y = \bigwedge_i (x_i \rightarrow y)$,

(9) $x \rightarrow (\bigvee_i y_i) \geq \bigvee_i (x \rightarrow y_i)$,

(10) $(\bigwedge_i x_i) \rightarrow y \geq \bigvee_i (x_i \rightarrow y)$,

(11) $(x \odot y) \rightarrow z = x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,

(12) $x \odot (x \rightarrow y) \leq y$ and $x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z)$,

(13) $(x \rightarrow y) \odot (z \rightarrow w) \leq (x \odot z) \rightarrow (y \odot w)$,

(14) $(x \to y) \odot (z \to w) \le (x \oplus z) \to (y \oplus w)$,

(15) $x \to y \le (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \le x \to z$,

(16) $x \odot y \odot (z \odot w) \le (x \odot z) \oplus (y \odot w)$.

(17) $x \to y = y^* \to x^*$.

Definition 3. [7-10] Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F, A) is called a *fuzzy soft set* over X, where $A \subset E$ and $F : A \to L^X$ is a mapping. We denote $S(X, A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 4. [7-10] Let (F, A) and (G, A) be two fuzzy soft sets over a common universe X:

(1) (F, A) is a fuzzy soft subset of (G, A), denoted by $(F, A) \le (G, A)$ if $F(\epsilon) \le G(\epsilon)$, for each $\epsilon \in A$.

(2) $(F, A) \wedge (G, A) = (F \wedge G, A)$ if $(F \wedge G)(\epsilon) = F(\epsilon) \wedge G(\epsilon)$ for each $\epsilon \in A$.

(3) $(F, A) \vee (G, A) = (F \vee G, A)$ if $(F \vee G)(\epsilon) = F(\epsilon) \vee G(\epsilon)$ for each $\epsilon \in A$.

(4) $(F, A) \odot (G, A) = (F \odot G, A)$ if $(F \odot G)(\epsilon) = F(\epsilon) \odot G(\epsilon)$ for each $\epsilon \in A$.

(5) $(F, A)^* = (F^*, A)$ if $F^*(\epsilon) = (F(\epsilon))^*$ for each $\epsilon \in A$.

(6) $(F, A) \oplus (G, A) = (F \oplus G, A)$ if $(F \oplus G)(\epsilon) = (F^*(\epsilon) \odot G^*(\epsilon))^*$ for each $\epsilon \in A$.

Definition 5. [10] A mapping $\xi : S(X, A) \times S(X, A) \to L$ is called a soft L-fuzzy semi-topogenous order on (X, A) if it satisfies the following axioms:

(ST1) $\xi((1_X, A), (1_X, A)) = \xi((0_X, A), (0_X, A)) = 1$.

(ST2) If $\xi((F, A), (G, A)) \ne 0$, then $(F, A) \le (G, A)$.

(ST3) If $(F_1, A) \le (F, A)$, $(G, A) \le (G_1, A)$, then $\xi((F, A), (G, A)) \le \xi((F_1, A), (G_1, A))$.

A mapping ξ is called a soft strong L-fuzzy semi-topogenous order on (X, A) if it satisfies (ST1), (ST3) and the following axiom.

(S) $\xi((F, A), (G, A)) \le S((F, A), (G, A))$ where

$$S((F, A), (G, A)) = \bigwedge_{a \in A} \bigwedge_{x \in X} (F(a)(x) \to G(a)(x)).$$

Remark 6. If ξ is a soft (resp. strong) L-fuzzy semi-topogenous order on (X, A). Define a mapping $\xi^s : S(X,A) \times S(X,A) \to L$ as $\xi^s((F,A),(G,A)) = \xi((G,A)^*,(F,A)^*)$. Then ξ^s is a soft (resp. strong) L-fuzzy semi-topogenous order on (X, A).

Definition 7. [10] A soft (resp. strong) L-fuzzy semi-topogenous order ξ is called:

(1) soft (resp. strong) L-fuzzy topogenous if (T)

$$\xi((F_1,A)\odot(F_2,A),(G_1,A)\odot(G_2,A)) \geq \xi((F_1,A),(G_1,A))\odot\xi((F_2,A),(G_2,A)).$$

(2) soft (resp. strong) L-fuzzy cotopogenous if (CT)

$$\xi((F_1,A)\oplus(F_2,A),(G_1,A)\oplus(G_2,A)) \geq \xi((F_1,A),(G_1,A))\odot\xi((F_2,A),(G_2,A)),$$

(3) soft (resp. strong) L-fuzzy bitopogenous if ξ are soft (resp. strong) L-fuzzy topogenous and soft (resp. strong) L-fuzzy cotopogenous.

Definition 8. [10] A soft (resp. strong) L-fuzzy topogenous (resp. cotopogenous) order ξ on (X, A) is said to be a soft (resp. strong) L-fuzzy topogenous (resp. cotopogenous) structure if $\xi \circ \xi \geq \xi$, where

$$(\xi \circ \xi)((F,A),(H,A)) \\ = \bigvee_{(G,A)\in S(X,A)} \xi((F,A),(G,A)) \odot \xi((G,A),(H,A)).$$

Definition 9. [10] A mapping $\mathcal{U} : S(X \times X, A) \to L$ is called a soft L-fuzzy pre-uniformity on X iff it satisfies the properties:

(SU1) There exists $(U,A) \in S(X \times X, A)$ such that $\mathcal{U}((U,A)) = 1$,

(SU2) If $(V,A) \leq (U,A)$, then $\mathcal{U}((V,A)) \leq \mathcal{U}((U,A))$,

(SU3) For every $(U,A),(V,A) \in S(X \times X, A)$,

$$\mathcal{U}((U,A) \odot (V,A)) \geq \mathcal{U}((U,A)) \odot \mathcal{U}((V,A)),$$

(SU4) If $\mathcal{U}((U,A)) \neq 0$, then $(1_\triangle, A) \leq (U,A)$, where

$$1_\Delta(x,y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y, \end{cases}$$

A soft L-fuzzy pre-uniformity $\mathcal{U}$ is called a soft L-fuzzy quasi-uniformity if

$$\text{(Q)}\ \ \mathcal{U}(U,A) \le \bigvee\{\mathcal{U}((V,A)) \odot \mathcal{U}((W,A)) \mid (V,A)\circ(W,A) \le (U,A)\},$$

where, for all $x, y \in X, a \in A$,

$$(V(a)\circ W(a))(x,y) = \bigvee_{z\in X}(V(a)(z,x)\odot W(a)(x,y)).$$

The triple $(X, A, \mathcal{U})$ is called a soft L-fuzzy pre-(resp. quasi-) uniform space.

Remark 10. Let $(X,\mathcal{U})$ be a soft L-fuzzy quasi-uniform space, then by (SU1) and (SU2), we have $\mathcal{U}(1_{X\times X}) = 1$ because $(U,A) \le (1_{X\times X}, A)$ for all $(U,A) \in S(X\times X, A)$.

Lemma 11. [7,10] For every $(F,A),(G,A) \in S(X,A)$, we define

$$(U_{F,G},A),(U^{-1}_{F,G},A) \in S(X\times X,A)$$

by

$$U_{F,G}(a)(x,y) = F(a)(x) \to G(a)(y),$$

$$U^{-1}_{F,G}(a)(x,y) = U_{F,G}(a)(y,x),$$

then we have the following statements:

(1) $(1_{X\times X},A) = (U_{0_X,0_X},A) = (U_{1_X,1_X},A)$,

(2) If $(F_1,A) \le (F_2,A)$ and $(G_1,A) \le (G_2,A)$, then $(U_{F_2,G_1},A) \le (U_{F_1,G_2},A)$,

(3) If $(F,A) \le (G,A)$, then $(1_\triangle,A) \le (U_{F,G},A)$,

(4) For every $(U_{F,G},A) \in S(X\times X,A)$ and $(H,A) \in S(X,A)$, we have $(U_{H,G},A)\circ(U_{F,H},A) \le (U_{F,G},A)$,

(5) $(U_{F_1,G_1},A)\odot(U_{F_2,G_2},A) \le (U_{F_1\odot F_2,G_1\odot G_2},A)$,

(6) $(U_{F_1,G_1},A)\odot(U_{F_2,G_2},A) \le (U_{F_1\oplus F_2,G_1\oplus G_2},A)$,

(7) $(U^{-1}_{F,G},A) = (U_{G^*,F^*},A)$,

(8) $(U^{-1}_{F_1,\odot F_2,G_1\odot G_2},A) = (U_{G_1^*\oplus G_2^*,F_1^*\oplus F_2^*},A)$,

(9) $(U^{-1}_{F_1\oplus F_2,G_1\oplus G_2},A) = (U_{G_1^*\odot G_2^*,F_1^*\odot F_2^*},A)$.

Lemma 12. [7,10] Let $(X, A, \mathcal{U})$ be a soft L-fuzzy quasi uniform space. For each $(U, A) \in S(X \times X, A)$ and $(F, A) \in S(X, A)$, the images $(U, A)[(F, A)]$, $(U, A)[[(F, A)]]$ of (F, A) with respect to (U, A) are defined by, for all $x \in X, a \in A$,

$$(U, A)[(F, A)](a)(x) = \bigvee_{y \in X} (F(a)(y) \odot U(a)(y, x)),$$

$$(U, A)[[(F, A)]](a)(x) = \bigvee_{y \in X} (F(a)(y) \odot U(a)(x, y)).$$

For each

$$(U, A), (V, A), (U_1, A), (U_2, A) \in S(X \times X, A)$$

and

$$(F, A), (G, A), (F_1, A), (F_2, A), (F_i, A) \in S(X, A),$$

we have:

(1) $(F, A) \leq (U, A)[(F, A)]$ and $(F, A) \leq (U, A)[[(F, A)]]$ for each $\mathcal{U}((U, A)) > 0$,

(2) $(U, A) \leq (U, A) \circ (U, A)$, for each $\mathcal{U}((U, A)) > 0$,

(3) $((V, A) \circ (U, A))[(F, A)] = (V, A)[(U, A)[(F, A)]]$, $((V, A) \circ (U, A))[[(F, A)]] = (V, A)[[(U, A)[[(F, A)]]]]$,

(4) $(U, A)[\bigvee_i (F_i, A)] = \bigvee_i (U, A)[(F_i, A)]$ and $(U, A)[[\bigvee_i (F_i, A)]] = \bigvee_i (U, A)[[(F_i, A)]]$,

(5) $((U_1, A) \odot (U_2, A))[(F_1, A) \odot (F_2, A)] \leq (U_1, A)[(F_1, A)] \odot (U_2, A)[(F_2, A)]$,

(6) $((U_1, A) \odot (U_2, A))[[(F_1, A) \odot (F_2, A)]] \leq (U_1, A)[[(F_1, A)]] \odot (U_2, A)[[(F_2, A)]]$,

(7) $((U_1, A) \odot ((U_2, A), A))[(F_1, A) \oplus (F_2, A)] \leq (U_1, A)[(F_1, A)] \oplus ((U_2, A), A)[(F_2, A)]$,

(8) $((U_1, A) \odot ((U_2, A), A))[[(F_1, A) \oplus (F_2, A)]] \leq (U_1, A)[[(F_1, A)]] \oplus ((U_2, A), A)[[(F_2, A)]]$.

(9) $(U_{F,G}, A) = \bigvee\{(W, A) \in S(X \times X, A) | (W, A)[(F, A)] \leq (G, A)\}$.

(10) $(U^{-1}_{F,G}, A) = \bigvee\{(W, A) \in S(X \times X, A) | (W, A)[[(F, A)]] \leq (G, A)\}$.

(11) $(U_{F,G}, A)[(F, A)] \leq (G, A)$ and $(U_{F,G}, A)[[(G, A)^*]] \leq (F, A)^*$. Moreover, $(U_{F,F}, A)[(F, A)] = (F, A)$ and $(U_{F,F}, A)[(F, A)^*] = (F, A)^*$.

Theorem 13. [10] *Let $(X, A, \mathcal{U})$ be a soft L-fuzzy quasi-uniform space. Define mappings $\xi^r_{\mathcal{U}}, \xi^l_{\mathcal{U}} : S(X, A) \times S(X, A) \to L$ by*

$$\xi^r_{\mathcal{U}}((F, A), (G, A)) = \bigvee\{\mathcal{U}((U, A)) \mid (U, A)[(F, A)] \leq (G, A)\},$$
$$\xi^l_{\mathcal{U}}((F, A), (G, A)) = \bigvee\{\mathcal{U}((U, A)) \mid (U, A)[[(F, A)]] \leq (G, A)\}.$$

Then $\xi^r_{\mathcal{U}}$ and $\xi^l_{\mathcal{U}}$ are soft L-fuzzy bitopogenous stuctures.

Theorem 14. [7] Let ξ be a soft L-fuzzy topogenous order on (X, A). Define $\mathcal{U}_\xi : S(X \times X, A) \to L$ by

$$\mathcal{U}_\xi((U, A)) = \bigvee\{\odot_{i=1}^n \xi((F_i, A), (G_i, A)) \mid \odot_{i=1}^n (U_{F_i,G_i}, A) \leq (U, A)\},$$

where $\bigvee$ is taken over every finite family $\{(U_{F_i,G_i}, A) \mid i = 1, 2, 3, ..., n\}$. Then:

(1) $\mathcal{U}_\xi((U_{\odot_{i=1}^n F_i, \odot_{i=1}^n G_i}, A)) = \xi(\odot_{i=1}^n (F_i, A), \odot_{i=1}^n (G_i, A))$.

(2) $\mathcal{U}_\xi$ is a soft L-fuzzy pre-uniformity on X.

(3) If ξ is a soft L-fuzzy topogenous structure on (X, A), then $\mathcal{U}_\xi$ is a soft L-fuzzy quasi-uniformity on X.

(4) $\xi^r_{\mathcal{U}_\xi} = \xi$ and $\xi^l_{\mathcal{U}_\xi} = \xi^s$.

Theorem 15. [7] Let ξ be a soft L-fuzzy cotopogenous order on (X, A). Define $\mathcal{U}_\xi : S(X \times X, A) \to L$ by

$$\begin{aligned}\mathcal{U}_\xi((U, A)) &= \bigvee\{\odot_{i=1}^n \xi((F_i, A), (G_i, A)) \mid \\ &\qquad \odot_{i=1}^n (U_{F_i,G_i}, A) \leq (U, A)\},\end{aligned}$$

where $\bigvee$ is taken over every finite family $\{(U_{F_i,G_i}, A) \mid i = 1, 2, 3, ..., n\}$. Then:

(1) $\mathcal{U}_\xi((U_{\oplus_{i=1}^n F_i, \oplus_{i=1}^n G_i}, A)) = \xi(\oplus_{i=1}^n (F_i, A), \oplus_{i=1}^n (G_i, A))$.

(2) $\mathcal{U}_\xi$ is a soft L-fuzzy pre-uniformity on X.

(3) If ξ is a soft L-fuzzy cotopogenous structure on (X, A), then $\mathcal{U}_\xi$ is a soft L-fuzzy quasi-uniformity on X.

(4) $\mathcal{U}_{\xi^s}((U, A)) = \mathcal{U}_\xi((U, A)^{-1})$ for all $(U, A) \in S(X \times X, L)$.

(5) $\xi^r_{\mathcal{U}_\xi} = \xi$ and $\xi^l_{\mathcal{U}_\xi} = \xi^s$.

3. The Relations between Soft L-Fuzzy Topogenous Orders and Soft L-Fuzzy Pre-Uniformities

Definition 16. A soft L-fuzzy pre-uniform structure $\mathcal{U}$ on X is said to be right compatible (resp. left compatible) with a soft L-fuzzy topogenous order ξ on (X, A) if $\xi^r_{\mathcal{U}} = \xi$ (resp. $\xi^l_{\mathcal{U}} = \xi$).

The class $\Pi^r(\xi)$ (resp. $\Pi^l(\xi)$) denotes the family of all soft L-fuzzy pre-uniformities which are right compatible(resp. left compatible) with a given soft L-fuzzy topogenous structure ξ.

Theorem 17. *Let ξ be a soft L-fuzzy topogenous order on (X, A) and the soft L-fuzzy topogenous orders $\xi^r_{\mathcal{U}_\xi}$ and $\xi^l_{\mathcal{U}_\xi}$ induced by $\mathcal{U}_\xi$. Then we have:*
(1) $\xi^r_{\mathcal{U}_\xi} = \xi$, *that is,* $\mathcal{U}_\xi \in \Pi^r(\xi)$.
(2) $\xi^l_{\mathcal{U}_\xi} = \xi^s$, *that is,* $\mathcal{U}_\xi \in \Pi^l(\xi^s)$.
(3) $\mathcal{U}_\xi$ *is the coarsest member of* $\Pi^r(\xi)$;*i.e.* $\mathcal{U}_\xi \le \mathcal{U}$ *for all* $\mathcal{U} \in \Pi^r(\xi)$.
(4) $\mathcal{U}_\xi$ *is the coarsest member of* $\Pi^l(\xi^s)$.

Proof (1) and (2)are easily proved from Theorems 14 (4) and 15 (5).

(3) By (1), we have that $\mathcal{U}_\xi$ is right compatible with ξ. Let $\mathcal{U}$ be an arbitrary member of $\Pi^r(\xi)$. We will show that $\mathcal{U}_\xi((U, A)) \le \mathcal{U}((U, A))$, for all $(U, A) \in S(X \times X, A)$.

Suppose that there exists $(U, A) \in S(X \times X, A)$ such that

$$\mathcal{U}_\xi((U, A)) \not\le \mathcal{U}((U, A)).$$

There exists a finite family $\{(U_{F_i,G_i}, A) \mid \odot_{i=1}^m (U_{F_i,G_i}, A) \le (U, A)\}$ such that

$$\odot_{i=1}^m \xi((F_i, A), (G_i, A)) \not\le \mathcal{U}((U, A)).$$

Since $\mathcal{U} \in \Pi^r(\xi)$, that is, $\xi(\lambda_i, \rho_i) = \xi^r_{\mathcal{U}}(\lambda_i, \rho_i)$ for $i = 1, ..., m$ and L is a complete residuated lattice, by the definition of $\xi^r_{\mathcal{U}}$, there exists $(V_i, A) \in S(X \times X, A)$ with $(V_i, A)[(F_i, A)] \le (G_i, A)$ such that

$$\odot_{i=1}^m \mathcal{U}((V_i, A)) \not\le \mathcal{U}((U, A)). \tag{I}$$

On the other hand, put $(V, A) = \odot_{i=1}^m (V_i, A)$. Since $(V_i, A)[(F_i, A)] \le (G_i, A)$, by the definition of (U_{F_i,G_i}, A), we have $(V_i, A) \le (U_{F_i,G_i}, A)$. It follows that

$$(V, A) = \odot_{i=1}^m (V_i, A) \le \odot_{i=1}^m (U_{F_i,G_i}, A) \le (U, A).$$

Hence

$$\begin{aligned} &\odot_{i=1}^m \mathcal{U}(V_i, A) \le \mathcal{U}(V, A) \\ &\le \mathcal{U}(\odot_{i=1}^m (U_{F_i,G_i}, A)) \le \mathcal{U}((U, A)). \end{aligned}$$

It is a contradiction for the equation (I).

(4) It is similarly proved as (3).

Theorem 18. *Let $(X, A, \mathcal{U})$ be a soft L-fuzzy pre-uniform space. Then:*
(1) $\mathcal{U}_{\xi^l_{\mathcal{U}}}((U, A)) \leq \mathcal{U}((U, A)^{-1})$ for all $(U, A) \in S(X \times X, A)$.
(2) $\mathcal{U}_{\xi^r_{\mathcal{U}}}((U, A)) \leq \mathcal{U}((U, A))$ for all $(U, A) \in S(X \times X, A)$.

Proof (1) Suppose that there exists $(U, A) \in S(X \times X, A)$ such that

$$\mathcal{U}_{\xi^l_{\mathcal{U}}}((U, A)) \not\leq \mathcal{U}((U, A)^{-1}).$$

From the definition of $\mathcal{U}_{\xi^l_{\mathcal{U}}}$, there exists a finite family $\{(U_{F_i,G_i}, A) \mid \odot_{i=1}^n (U_{F_i,G_i}, A) \leq (U, A)\}$ such that

$$\odot_{i=1}^n \xi^l_{\mathcal{U}}((F_i, A), (G_i, A)) \not\leq \mathcal{U}((U, A)^{-1}).$$

From the definition of $\xi^l_{\mathcal{U}}$, for each $i \in \{1, 2, ..., n\}$, there exists $(V_i, A) \in S(X \times X, A)$ with $(V_i, A)[[(F_i, A)]] \leq (G_i, A)$ such that

$$\odot_{i=1}^n \mathcal{U}((V_i, A)) \not\leq \mathcal{U}((U^{-1}, A)).$$

Let $(V, A) = \odot_{i=1}^n (V_i, A)$ be given. Since $(V_i, A)[[(F_i, A)]] \leq (G_i, A)$, by the definition of $(U^{-1}_{F_i,G_i}, A)$, we have $(V_i, A) \leq (U^{-1}_{F_i,G_i}, A)$. Hence $(U^{-1}_{F_i,G_i}, A) \geq \odot_{i=1}^n (U^{-1}_{F_i,G_i}, A) \geq \odot_{i=1}^n (V_i, A) = (V, A)$ and

$$\mathcal{U}((U^{-1}, A)) \geq \mathcal{U}(\odot_{i=1}^n (U^{-1}_{F_i,G_i}, A)) \geq \odot_{i=1}^n \mathcal{U}((V_i, A)).$$

It is a contradiction. Therefore, $\mathcal{U}_{\xi^l_{\mathcal{U}}}((U, A)) \leq \mathcal{U}((U^{-1}, A))$.

(2) It is similarly proved as (1).

Definition 19. [10] Let $S(X, A)$ and $S(Y, B)$ be the families of all fuzzy soft sets over X and Y, respectively. The mapping $f_\phi : S(X, A) \to S(Y, B)$ is a soft mapping where $f : X \to Y$ and $\phi : A \to B$ are mappings.

(1) The image of $(F, A) \in S(X, A)$ under the mapping f_ϕ is denoted by $f_\phi((F, A)) = (f_\phi(F), B)$ where

$$f_\phi(F)(b) = \begin{cases} \bigvee_{a \in \phi^{-1}(\{b\})} f^{\to}(F(a)), & \text{if } \phi^{-1}(\{b\}) \neq \emptyset, \\ \overline{0}, & \text{otherwise.} \end{cases}$$

(2) The inverse image of $(G, B) \in S(Y, B)$ under the mapping f_ϕ is denoted by $f_\phi^{-1}((G, B)) = (f_\phi^{-1}(G), A)$ where

$$f_\phi^{-1}(G)(a)(x) = f^{\leftarrow}(G(\phi(a)))(x), \ \forall a \in A, x \in X.$$

(3) The soft mapping $f_\phi : S(X,A) \to S(Y,B)$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 20. [10] Let $f_\phi : S(X,A) \to S(Y,B)$ be a soft mapping. Then we have the following properties. For $(F,A),(F_i,A) \in S(X,A)$ and $(G,B),(G_i,B) \in S(Y,B)$,

(1) $(G,B) \geq f_\phi(f_\phi^{-1}((G,B)))$ with equality if f is surjective,

(2) $(F,A) \leq f_\phi^{-1}(f_\phi((F,A)))$ with equality if f is injective,

(3) $f_\phi^{-1}((G,B)^*) = (f_\phi^{-1}((G,B)))^*$,

(4) $f_\phi^{-1}(\bigvee_{i\in I}(G_i,B)) = \bigvee_{i\in I} f_\phi^{-1}((G_i,B))$,

(5) $f_\phi^{-1}(\bigwedge_{i\in I}(G_i,B)) = \bigwedge_{i\in I} f_\phi^{-1}((G_i,B))$,

(6) $f_\phi(\bigvee_{i\in I}(F_i,A)) = \bigvee_{i\in I} f_\phi((F_i,A))$,

(7) $f_\phi(\bigwedge_{i\in I}(F_i,A)) \leq \bigwedge_{i\in I} f_\phi((F_i,A))$ with equality if f is injective,

(8) $f_\phi^{-1}((G_1,B) \odot (G_2,B)) = f_\phi^{-1}((G_1,B)) \odot f_\phi^{-1}((G_2,B))$,

(9) $f_\phi^{-1}((G_1,B) \oplus (G_2,B)) = f_\phi^{-1}((G_1,B)) \oplus f_\phi^{-1}((G_2,B))$,

(10) $f_\phi((F_1,A) \odot (F_2,A)) \leq f_\phi((F_1,A)) \odot f_\phi((F_2,A))$ with equality if f is injective,

(11) $f_\phi((F_1,A) \oplus (F_2,A)) \leq f_\phi((F_1,A)) \oplus f_\phi((F_2,A))$ with equality if f is injective.

Theorem 21. [10] Let $(X,A,\mathcal{U})$ and $(Y,B,\mathcal{V})$ be soft L-fuzzy quasi-uniform spaces. Let $f_\phi : (X,A,\mathcal{U}) \to (Y,B,\mathcal{V})$ be a uniformly continuous soft map. Then:

(1) $f_\phi : (X,A,\xi^r_{\mathcal{U}}) \to (Y,B,\xi^r_{\mathcal{V}})$ is a topogenous continuous soft map.

(2) $f_\phi : (X,A,\xi^l_{\mathcal{U}}) \to (Y,B,\xi^l_{\mathcal{V}})$ is a topogenous continuous soft map.

Theorem 22. Let (X,A,ξ_X) and (Y,B,ξ_Y) be soft L-fuzzy topogenous ordered sets. Then a mapping $f_\phi : (X,\xi_X,A) \to (Y,\xi_Y,B)$ is a topogenous continuous soft map iff the mapping $f_\phi : (X,A,\mathcal{U}_{\xi_X}) \to (Y,A,\mathcal{U}_{\xi_Y})$ is an uniformly continuous soft map.

Proof. Since $f_\phi : (X,\xi_X,A) \to (Y,\xi_Y,B)$ is a topogenous continuous soft map, then

$$
\begin{aligned}
&(f\times f)_\phi^{-1}((U_{F,G},B))(a)(x,y) = U_{F,G}(\phi(a))(f(x),f(y))\\
&= F(\phi(a))(f(x)) \to G(\phi(a))(f(y)) = f_\phi^{-1}(F)(a)(x) \to f_\phi^{-1}(G)(a)(y)\\
&= (U_{f_\phi^{-1}(F),f_\phi^{-1}(G)},A)(a)(x,y)
\end{aligned}
$$

$$\mathcal{U}_{\xi_X}((f\times f)_\phi^{-1}((V,B)))$$
$$\geq \bigvee\{\odot_{i=1}^n \xi_X(f_\phi^{-1}((F_i,B)), f_\phi^{-1}((G_i,B))) \mid$$
$$\odot_{i=1}^n (U_{f_\phi^{-1}(F_i),f_\phi^{-1}(G_i)}, A) \leq (f\times f)_\phi^{-1}((V,B))\}$$
$$\geq \bigvee\{\odot_{i=1}^n \xi_Y((F_i,B),(G_i,B)) \mid \odot_{i=1}^n((U_{F_i,G_i},B)) \leq (V,B)\}$$
$$= \mathcal{U}_{\xi_Y}((V,B)).$$

Conversely, let $f:(X,\mathcal{U}_{\xi_X}) \to (Y,\mathcal{U}_{\xi_Y})$ is an uniformly continuous soft map. Since $\xi^r_{\mathcal{U}_{\xi_X}} = \xi_X$ and $\xi^r_{\mathcal{U}_{\xi_Y}} = \xi_Y$, by Theorems 15(4) and 21, the result holds.

Example 23. Let $U = \{h_i \mid i = \{1,...,6\}\}$ with h_i=house and $E = \{e,b,w,c,i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surroundings.

Define a binary operation $\odot$ on $[0,1]$ by

$$x\odot y = \max\{0, x+y-1\},\ x\to y = \min\{1-x+y, 1\}$$

$$x\oplus y = \min\{1, x+y\},\ x^* = 1-x$$

Then $([0,1],\wedge,\to,0,1)$ is a complete residuated lattice (ref.[4.5]). Let $A = \{b,c,i\}\subset E$ and $X = \{h_1,h_4,h_5,h_6\}$. Put (H,A) be a fuzzy soft set as follow:

(H,A)	h_1	h_4	h_5	h_6
b	0.5	0.6	0.2	0.6
c	0.1	0.5	0.5	0.6
i	0.4	0.6	0.6	0.5

$(H,A)\odot(H,A)$	h_1	h_4	h_5	h_6
b	0.0	0.2	0.0	0.2
c	0.0	0.0	0.0	0.2
i	0.0	0.2	0.2	0.0

(H^*,A)	h_1	h_4	h_5	h_6
b	0.5	0.4	0.8	0.4
c	0.9	0.5	0.5	0.4
i	0.6	0.4	0.4	0.5

$(H^*,A)\oplus(H^*,A)$	h_1	h_4	h_5	h_6
b	1.0	0.8	1.0	0.8
c	1.0	1.0	1.0	0.8
i	1.0	0.8	0.8	1.0

(1) Define a soft L-fuzzy topogenous order $\xi : S(X,A) \times S(X,A) \to L$ as follows

$$\xi((F,A),(G,A)) = \begin{cases} 1, & \text{if } (F,A) = (\overline{0},A) \text{ or } (G,A) = (\overline{1},A) \\ 0.6, & \text{if } (F,A) \le (H,A) \le (G,A), \\ & (F,A) \not\le (H,A) \odot (H,A) \\ 0.3, & \text{if } (\overline{0},A) \ne (F,A) \le (H,A) \odot (H,A) \\ & \le (G,A), (H,A) \not\le (G,A), \\ 0, & \text{otherwise}, \end{cases}$$

We obtain $(U_{H,H},A), (U_{H\odot H,H\odot H},A) \in S(X\times X,A)$ such that, for $a \in A$, $U_{H,H}(a) \in L^{X\times X}$ with $U_{H,H}(a)(x,y) = H(a)(x) \to H(a)(y)$,

$$U_{H,H}(b) = \begin{pmatrix} 1 & 1 & 0.7 & 1 \\ 0.9 & 1 & 0.6 & 1 \\ 1 & 1 & 1 & 1 \\ 0.9 & 1 & 0.6 & 1 \end{pmatrix}, U_{H,H}(c) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0.6 & 1 & 1 & 1 \\ 0.6 & 1 & 1 & 1 \\ 0.5 & 0.9 & 0.9 & 1 \end{pmatrix}$$

$$U_{H,H}(i) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0.8 & 1 & 1 & 0.9 \\ 0.8 & 1 & 1 & 0.9 \\ 0.9 & 1 & 1 & 1 \end{pmatrix}, U_{H\odot H,H\odot H}(b) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0.8 & 1 & 0.8 & 1 \\ 1 & 1 & 1 & 1 \\ 0.8 & 1 & 0.8 & 1 \end{pmatrix}$$

$$U_{H\odot H,H\odot H}(c) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0.8 & 0.8 & 0.8 & 1 \end{pmatrix}, U_{H\odot H,H\odot H}(i) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0.8 & 1 & 1 & 0.8 \\ 0.8 & 1 & 1 & 0.8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

By Theorem 14, we obtain a soft L-fuzzy pre-uniformity $\mathcal{U}_\xi : S(X\times X,A) \to L$ as follows

$$\mathcal{U}_\xi((U,A)) = \begin{cases} 1, & \text{if } (U,A) = (1_{X\times X},A), \\ 0.6, & \text{if } (U_{H,H},A) \le (U,A) \ne (1_{X\times X},A), \\ 0.3, & \text{if } (U_{H\odot H,H\odot H},A) \le (U,A) \\ & (U,A) \not\ge (U_{H,H},A), \\ 0.2, & \text{if } (U_{H,H},A) \odot (U_{H,H},A) \le (U,A), \\ & (U,A) \not\ge (U_{H\odot H,H\odot H},A), \\ 0, & \text{otherwise}. \end{cases}$$

Since $U_{H,H}(H,A) = (H,A)$ and $(U_{H\odot H,H\odot H},A)(H\odot H,A) = (H\odot H,A)$ from Lemma 12 (11), by Theorems 13 and 14(4), we have $\xi^r_{\mathcal{U}_\xi} = \xi$, that is, $\mathcal{U}_\xi \in \Pi^r(\xi)$.

(2) By Remark 6 and (1), we obtain a soft L-fuzzy cotopogenous order $\xi^s : S(X,A) \times S(X,A) \to L$ as follows

$$\xi^s((F,A),(G,A)) = \begin{cases} 1, & \text{if } (F,A) = (\overline{0},A) \text{ or } (G,A) = (\overline{1},A) \\ 0.6, & \text{if } (F,A) \leq (H,A)^* \leq (G,A), \\ & (G,A) \not\geq (H,A)^* \oplus (H,A)^* \\ 0.3, & \text{if } \ (F,A) \leq (H,A)^* \oplus (H,A)^* \\ & \leq (G,A) \neq (\overline{1},A), (F,A) \not\leq (H,A)^*, \\ 0, & \text{otherwise}, \end{cases}$$

We obtain $(U_{H,H},A), (U_{H\odot H,H\odot H},A) \in S(X \times X, A)$ such that, for $a \in A$, $U_{H,H}(a) \in L^{X\times X}$ with $U_{H,H}(a)(x,y) = H(a)(x) \to H(a)(y)$,

$$U_{H^*,H^*}(b) = \begin{pmatrix} 1 & 0.9 & 1 & 0.9 \\ 1 & 1 & 1 & 1 \\ 0.7 & 0.6 & 1 & 0.6 \\ 1 & 1 & 1 & 1 \end{pmatrix}, U_{H^*,H^*}(c) = \begin{pmatrix} 1 & 0.6 & 0.6 & 0.5 \\ 1 & 1 & 1 & 0.9 \\ 1 & 1 & 1 & 0.9 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$U_{H^*,H^*}(i) = \begin{pmatrix} 1 & 0.8 & 0.8 & 0.9 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0.9 & 0.9 & 1 \end{pmatrix}, U_{H^*\oplus H^*,H^*\oplus H^*}(b) = \begin{pmatrix} 1 & 0.8 & 1 & 0.8 \\ 1 & 1 & 1 & 1 \\ 1 & 0.8 & 1 & 0.8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$U_{H^*\oplus H^*,H^*\oplus H^*}(c) = \begin{pmatrix} 1 & 1 & 1 & 0.8 \\ 1 & 1 & 1 & 0.8 \\ 1 & 1 & 1 & 0.8 \\ 1 & 1 & 1 & 1 \end{pmatrix}, U_{H^*\oplus H^*,H^*\oplus H^*}(i) = \begin{pmatrix} 1 & 0.8 & 0.8 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0.8 & 0.8 & 1 \end{pmatrix}$$

By Theorem 15, we obtain a soft L-fuzzy pre-uniformity $\mathcal{U}_{\xi^s} : S(X \times X, A) \to L$ as follows

$$\mathcal{U}_{\xi^s}((U,A)) = \begin{cases} 1, & \text{if } (U,A) = (1_{X\times X},A), \\ 0.6, & \text{if } (U_{H^*,H^*},A) \leq (U,A) \neq (1_{X\times X},A), \\ 0.3, & \text{if } (U_{H^*\oplus H^*,H^*\oplus H^*},A) \leq (U,A) \\ & (U,A) \not\geq (U_{H^*,H^*},A), \\ 0.2, & \text{if } (U_{H^*,H^*},A) \odot (U_{H^*,H^*},A) \leq (U,A), \\ & (U,A) \not\geq (U_{H^*\oplus H^*,H^*\oplus H^*},A), \\ 0, & \text{otherwise}. \end{cases}$$

Since $U_{H^*,H^*}(H^*,A) = (H^*,A)$ and $(U_{H^*\oplus H^*,H^*\oplus H^*},A)(H^* \oplus H^*,A) = (H^* \oplus H^*,A)$ from Lemma 12 (11), by Theorems 13 and 15(5), we have $\xi^r_{\mathcal{U}_{\xi^s}} = \xi^s$, that is, $\mathcal{U}_{\xi^s} \in \Pi^r(\xi)$. Moreover, by Theorems 13 and 14(4), we have $\xi^l_{\mathcal{U}_\xi} = \xi^s$, that is, $\mathcal{U}_\xi \in \Pi^l(\xi^s)$.

(3) Let $B = \{e, b\} \subset E$ and $Y = \{h_i \mid i = \{1,2,3\}\}$ be given. Put $V, V \odot V \in S(Y \times Y, B)$ as

$V(e)$	h_1	h_2	h_3
h_1	1	0.6	0.5
h_2	0.1	1	0.5
h_3	0.4	0.6	1

$V(b)$	h_1	h_2	h_3
h_1	1	0.5	0.3
h_2	0.7	1	0.5
h_3	0.6	0.6	1

$(V \odot V)(e)$	h_1	h_2	h_3
h_1	1	0.2	0
h_2	0	1	0
h_3	0	0.2	1

$(V \odot V)(b)$	h_1	h_2	h_3
h_1	1	0	0
h_2	0.4	1	0
h_3	0.2	0.2	1

We define $\mathcal{U}_Y : S(Y \times Y, B) \to [0,1]$ as follows:

$$\mathcal{U}_Y((U,A)) = \begin{cases} 1, & \text{if } (U,A) = (1_{Y\times Y}, B) \\ 0.6, & \text{if}(U,A) \geq (V,B), \\ 0.3, & \text{if}(U,A) \geq (V \odot V, B), (U,A) \not\geq (V,B), \\ 0, & \text{otherwise.} \end{cases}$$

Since $(V,B) \circ (V,B) = (V,B)$ and $(V \odot V, B) \circ (V \odot V, B) = (V \odot V, B)$, $\mathcal{U}_Y$ is a soft L-fuzzy quasi-uniformity on Y.

From Theorem 13, we obtain a soft L-fuzzy topogenous order $\xi^r_{\mathcal{U}_Y} : S(Y,B) \times S(Y,B) \to L$ as follows

$$\xi_{\mathcal{U}_Y}((F,B),(G,B)) = \begin{cases} 1, & \text{if } [(1_{Y\times Y}, B)](F,B) \leq (G,B), \\ 0.6, & \text{if } [(V,B)](F,B) \leq (G,B), \\ & [(V \odot V, B)](F,B) \not\leq [(V,B)](F,B), \\ 0.3, & \text{if } \ [(V \odot V, B)](F,B) \leq (G,B) \\ 0, & \text{otherwise,} \end{cases}$$

From Theorem 14, we obtain $\mathcal{U}_{\xi^r_{\mathcal{U}_Y}} : S(Y \times Y, B) \to [0,1]$ as follows:

$$\mathcal{U}_{\xi^r_{\mathcal{U}_Y}}((U,B)) = \begin{cases} 1, & \text{if } (U,B) = (1_{Y\times Y}, B) \\ 0.6, & \text{if}(U,B) \geq (U_{V_F,V_F}, B), \\ 0.3, & \text{if}(U,B) \geq (U_{(V\odot V)_F,(V\odot V)_F}, B), (U,B) \not\geq (V,B), \\ 0, & \text{otherwise,} \end{cases}$$

where $[(V,B)](F,B) = (V_F, B)$ and $[(V \odot V, B)](F,B) = ((V \odot V)_F, B)$. Since

$$\begin{aligned} &\textstyle\bigvee_{x\in X}(V(b)(x,y) \odot F(b)(x)) \to \bigvee_{x\in X}(V(b)(x,z) \odot F(b)(x)) \\ &\geq \textstyle\bigwedge_{x\in X}((V(b)(x,y) \odot F(b)(x)) \to (V(b)(x,z) \odot F(b)(x))) \\ &\geq \textstyle\bigwedge_{x\in X}(V(b)(x,y) \to V(b)(x,z)) \geq V(b)(y,z), \end{aligned}$$

$U_{V_F,V_F} \geq V$ and $U_{(V\odot V)_F,(V\odot V)_F} \geq V \odot V$. Hence $\mathcal{U}_{\xi^r_{\mathcal{U}_Y}} \leq \mathcal{U}_Y$.

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