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(K, E)-SOFT TOPOLOGIES AND L-FUZZY (K, E)-SOFT NEIGHBORHOOD SYSTEMS

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Abstract: In this paper, we define a (K, E)-soft topology in stsc-quantales. We investigate the relations between (K, E)-soft topologies and L-fuzzy (K, E)-soft neighborhood systems. We give their examples.

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1. Introduction

Molodtsov [14] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. Maji et al. [11,12] gave the first practical application of soft sets in decision making problems. Many researchers have contributed towards the algebraic structure of soft set theory [1-5,7]. In 2011, Shabir and Naz [22] initiated the study of

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soft topological spaces. They defined soft topology on the collection of soft sets over X and established their several properties. Aygünoglu et.al [2] introduced the concept of (K, E)-soft topology in the sense of Šostak [9]. Cetkin et.al [3] studied (K, E)-soft proximities and discuss their properties.

Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic and decision rules in complete residuated lattices. Höhle [9] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV-algebra). It has developed in many directions [17-19]. Ramadan et al. [18] define the the concept of L- fuzzy soft topogenous orders, L-fuzzy soft uniform spaces, L- fuzzy soft topological spaces in strictly two sided commutative quantales and investigated the relation between them.

In this paper, we define a (K, E)-soft topology in stsc-quantales. We investigate the relations between (K, E)-soft topologies and L-fuzzy (K, E)-soft neighborhood systems. We give their examples.

2. Preliminaries

Let $L = (L, \leq, \vee, \wedge, 0, 1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L.

Definition 1. [8,9,18] A complete lattice $(L, \leq, \odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) iff it satisfies the following properties.

(L1) $(L, \odot)$ is a commutative semigroup,

(L2) $x = x \odot 1$, for each $x \in L$ and 1 is the universal upper bound,

(L3) $\odot$ is distributive over arbitrary joins, i.e. $(\bigvee_i x_i) \odot y = \bigvee_i (x_i \odot y)$.

There exists a further binary operation $\rightarrow$ (called the implication operator or residuated) satisfying the following condition

$$x \rightarrow y = \bigvee \{z \in L | x \odot z \leq y\}.$$

Then it satisfies Galois correspondence; i.e, $(x \odot z) \leq y$ iff $z \leq (x \rightarrow y)$.

In this paper, we always assume that $(L, \leq, \odot, \rightarrow, ^*)$ is a stsc-quantales with an order reversing involution * which is defined $x^* = x \rightarrow 0$ unless otherwise specified.

Remark 2. Every completely distributive lattice $(L, \leq, \wedge, \vee, ^*)$ with order reversing involution * is a stsc-quantale $(L, \leq, \odot = \wedge, ^*)$ with a strong negation *.

Lemma 3. [8,9,18] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \leq y$ iff $x \to y = 1$.

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(7) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(8) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(10) $x \odot y = (x \to y^*)^*$ and $x \oplus y = x^* \to y$,

(11) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(12) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(13) $x \to y = y^* \to x^*$.

(14) $\bigvee_{i \in \Gamma} x_i \to \bigvee_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$ and $\bigwedge_{i \in \Gamma} x_i \to \bigwedge_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 4. [4] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f_e^* = (f_e)^*$, for each $e \in E$.

(6) f is called a null L- fuzzy soft set and is denoted by 0_X , if $f_e(x) = 0$, for each $e \in E$, $x \in X$.

(7) f is called an absolute L- fuzzy soft set and is denoted by 1_X , if $f_e(x) = 1$, for each $e \in E$, $x \in X$ and $(1_x)_e(x) = 1$.

Definition 5. [4] Let $\varphi : X \to Y$ and $\psi : E_1 \to E_2$ be two mappings, where E_1 and E_2 are parameters sets for the crisp sets X and Y, respectively.

Then $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ is called a fuzzy soft mapping.

(1) For $f \in (L^X)^{E_1}$, the image of f under the fuzzy soft mapping φ_ψ defined by, $\forall k \in K, \forall y \in Y$,

$$\varphi_\psi(f)_{e_2}(y) = \bigvee_{x\in\varphi^{-1}(\{y\})} \Big(\bigvee_{e_1\in\psi^{-1}(\{e_2\})} f_{e_1}(x)\Big)$$

(2) For $f \in (L^X)^{E_1}$, the pre-image of g defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E, \forall x \in X.$$

(3) The soft mapping $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 6. [10] Let $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ be a soft mapping. Then we have the following properties. For $f, f_i \in (L^X)^{E_1}$ and $g, g_i \in (L^Y)^{E_2}$,

(1) $g \sqsupseteq \varphi_\psi(\varphi_\psi^{-1}(g))$ with equality if φ_ψ is surjective,

(2) $f \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(f))$ with equality if φ_ψ is injective,

(3) if φ_ψ is injective,

$$\varphi_\psi(f)_{e_2}(y) = \begin{cases} f_{e_1}(x), & \text{if } x \in \varphi^{-1}(y), e_1 \in \psi^{-1}(e_2) \\ 0, & \text{otherwise,} \end{cases}$$

(4) $\varphi_\psi^{-1}(g^*) = (\varphi_\psi^{-1}(g))^*$,

(5) $\varphi_\psi^{-1}(\bigvee_{i\in I} g_i) = \bigvee_{i\in I} \varphi_\psi^{-1}(g_i)$,

(6) $\varphi_\psi^{-1}(\bigwedge_{i\in I} g_i) = \bigwedge_{i\in I} \varphi_\psi^{-1}(g_i)$,

(7) $\varphi_\psi(\bigvee_{i\in I} f_i) = \bigvee_{i\in I} \varphi_\psi(f_i)$,

(8) $\varphi_\psi(\bigwedge_{i\in I} f_i) \sqsubseteq \bigwedge_{i\in I} \varphi_\psi(f_i)$ with equality if φ_ψ is injective,

(9) $\varphi_\psi^{-1}(g_1 \odot g_2) = \varphi_\psi^{-1}(g_1) \odot \varphi_\psi^{-1}(g_2)$,

(10) $\varphi_\psi(f_1 \odot f_2) \sqsubseteq \varphi_\psi(f_1) \odot \varphi_\psi(f_2)$ with equality if φ_ψ is injective,

Definition 7. [2,16] A set $\tau\{\tau_k \subset P((L^X)^E) \mid k \in K\}$ is called a (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(O1) $0_X, 1_X \in \tau_k$,

(O2) If $f, g \in \tau_k$, then $(f \odot g) \in \tau_k$.

(O3) If $f_i \in \tau_k$, $\bigsqcup_{i\in I} f_i \in \tau_k$.

The pair (X, τ) is called a (K, E)-soft topological space. Let (X, τ^1) be a (K_1, E_1)-soft topological space and (Y, τ^2) be a (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X, τ^1) into (Y, τ^2) is called soft continuous if $\varphi_\psi^{-1}(f) \in (\tau^1)_k \;\; \forall f \in (\tau^2)_{\eta(k)}, k \in K_1$.

Definition 8. [19] An L-fuzzy (K,E)-soft neighborhood system on X is a set $N=\{N^x \mid x\in X\}$ of mappings $N^x : K\to L^{(L^X)^E}$ such that for each $k\in K$:

(SN1) $N^x_k(1_X)=1$ and $N^x_k(0_X)=0$,

(SN2) $N^x_k(f\odot g)\geq N^x_k(f)\odot N_x(g)$ for each $f,g\in (L^X)^E$,

(SN3) If $f\sqsubseteq g$, then $N^x_k(f)\leq N^x_k(g)$,

(SN4) $N^x_k(f)\leq f_e(x)$ for all $f\in (L^X)^E$ and $e\in E$.

(SN5) $N^x_k(f)\leq \bigvee\{N^x_k(g)\mid g_e(y)\sqsubseteq N^y_k(f),\ \forall\, y\in X, e\in E\}$.

The previous axiom can be reformulated in the following way

(SN5) $\forall\, f\in (L^X)^E$ and $x\in X$, $N^x_k(f)\leq N^x_k(N^-_k(f))$, where $N^-_k(f)\in (L^X)^E$ is defined by

$$(N^-_k(f))_e(y)=N^y_k(f)\ \ \forall\, y\in Y, e\in E.$$

An L-fuzzy (K,E)-soft neighborhood system is called stratified if

(SR) $N^x_k(\alpha\odot f)\geq \alpha\odot N^x_k(f)$ for all $f\in (L^X)^E$ and $\alpha\in L$.

The pair (X,N) is called an L-fuzzy (K,E)-soft neighborhood space.

Let (X,N) be an L-fuzzy (K_1,E_1)-soft neighborhood space and (Y,M) be an L-fuzzy (K_2,E_2)-soft neighborhood space. Let $\varphi : X\to Y$, $\psi : E_1\to E_2$ and $\eta : K_1\to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X,N) into (Y,M) is called L-fuzzy soft N-continuous at every $x\in X$ if $M^{\phi(x)}_{\eta(k)}(f)\leq N^x_k(\varphi^{-1}_\psi(f))\ \ \forall\, f\in (L^Y)^{E_2}, k\in K_1$.

Lemma 9. [19] Define a binary mapping $S : (L^X)^E\times (L^X)^E\to L$ by

$$S(f,g)=\bigwedge_{x\in X}\bigwedge_{e\in E}(f_e(x)\to g_e(x))\ \ \forall\, f,g\in (L^X)^E,\ \ \forall\, e\in E.$$

Then $\forall f,g,h,m,n\in (L^X)^E$ the following statements hold.

(1) $f\sqsubseteq g$ iff $S(f,g)=1$.

(2) If $f\sqsubseteq g$, then $S(h,f)\leq S(h,g)$ and $S(f,h)\geq S(g,h)$.

(3) $S(f,h)\odot S(h,g)\leq S(f,g)$. Moreover, $\bigvee_{h\in (L^X)^E}(S(f,h)\odot S(h,g))=S(f,g)$

(4) $S(f,g)\odot S(m,n)\leq S(f\odot m, g\odot n)$.

(5) If $\varphi_\psi : (X,E)\to (Y,F)$ is a fuzzy soft mapping, then $S(p,q)\leq S(\varphi^{-1}_\psi(p),\varphi^{-1}_\psi(q))$, for each $p,q\in (L^Y)^F$.

3. (K,E)-Soft Topologies and L-Fuzzy (K,E)-Soft Neighborhood Systems

Theorem 10. *Let (X,τ) be a (K,E)-soft topological space. Define a map $N^\tau : X \to L^{(L^X)^E}$ by*

$$(N^\tau)^x_k(f) = \bigvee\{\bigwedge_{e\in E} g_e(x) \mid g \sqsubseteq f,\ g \in \tau_k\}.$$

Then the following properties hold.

(1) (X, N^τ) is a L-fuzzy (K,E)-soft neighborhood space.

(2) If τ is enriched, then N^τ is stratified and

$$(N^\tau)^x_k(f) = \bigvee_{g\in\tau}(\bigwedge_{e\in E} g_e(x) \odot S(g,f).$$

Proof. (1) (SN1) Since $1_X, 0_X \in \tau_k$, $(N^\tau)^x_k(1_X) = 1$ and $(N^\tau)^x_k(0_X) = 0$.

(SN2)

$$\begin{array}{l}(N^\tau)^x_k(f) \odot (N^\tau)^x_k(g)\\ = \big(\bigvee\{\bigwedge_{e\in E}(f_1)_e(x) \mid f_1 \sqsubseteq f,\ f_1 \in \tau_k\}\big)\\ \odot\big(\bigvee\{\bigwedge_{e\in E}(g_1)_e(x) \mid g_1 \sqsubseteq g,\ g_1 \in \tau_k\}\big)\\ \le \bigvee\{\bigwedge_{e\in E}(f_1\odot g_1)_e(x) \mid f_1\odot g_1 \sqsubseteq f\odot g,\ f_1\odot g_1 \in \tau_k\}\\ \le (N^\tau)^x_k(f\odot g).\end{array}$$

(SN3-5) follow from the definition of N^τ.

(SN6) Put $N^\tau_-(f) = \bigvee\{g \mid g \sqsubseteq f,\ g\in\tau_k\}$ with $N^\tau_-(x) = (N^\tau)^x_k$. Then $N^\tau_-(f)\in\tau_k$. By (SN3) and the definition of N^τ,

$$(N^\tau)^x_k(N^\tau_-(f)) = (N^\tau)^x_k(f).$$

$$\begin{array}{l}(N^\tau)^x_k(f) = (N^\tau)^x_k(N^\tau_-(f))\\ \le \bigvee\{(N^\tau)^x_k(g) \mid g_e(y) \le N^\tau_y(f)\}.\end{array}$$

Thus (X, N^τ) is an L-neighborhood space.

(2)

$$\begin{array}{l}\alpha\odot(N^\tau)^x_k(f) = \alpha\odot\bigvee\{\bigwedge_{e\in E} g_e(x) \mid g\sqsubseteq f,\ g\in\tau_k\}\\ \le \bigvee\{\bigwedge_{e\in E}(\alpha\odot g_e)(x) \mid \alpha\odot g\sqsubseteq \alpha\odot f,\ \alpha\odot g\in\tau\} \le (N^\tau)^x_k(\alpha\odot f).\end{array}$$

Put $\gamma(x) = \bigvee_{g\in\tau_k}(\bigwedge_{e\in E} g_e(x)\odot S(g,f))$. Let g with $g\sqsubseteq f$ and $g\in\tau_k$. Then $g_e(x)\odot S(g,f) = g_e(x)\odot 1 = g_e(x)$. Thus $\bigwedge_{e\in E} g_e(x) \le \gamma(x)$. Therefore $(N^\tau)^x_k(f) \le \gamma(x)$.

Let $g_e(x)\odot S(g,f)$ with $g\in\tau_k$. Since τ is enriched, $g\odot S(g,f)\in\tau_k$ and $g_e(x)\odot S(g,f) \le g_e(x)\odot(g_e(x)\to f_e(x)) \le f_e(x)$. Then $\gamma(x) \le (N^\tau)^x_k(f)$.

Theorem 11. *Let (X, N) be an L-fuzzy (K, E)-soft neighborhood space. Define $\tau_k^N \subset (L^X)^E$ as follows*

$$\tau_k^N = \{f \in (L^X)^E \mid f_e(x) = N_k^x(f), \forall x \in X, e \in E\}.$$

Then,

(1) τ^N is a (K, E)-soft topology on X,

(2) If N is stratified, then τ^N is an enriched (K, E)-soft topology.

(3) $N \leq N^{\tau^N}$. If $E = \{e\}$, then $N = N^{\tau^N}$.

(4) If (X, τ) is a (K, E)-soft topological space and $E = \{e\}$, then $\tau = \tau^{N^\tau}$.

Proof. (1) (O1) Since $N_k^x(1_X) = 1$ and $N_k^x(0_X) = 0$, we have $1_X, 0_X \in \tau_k^N$.

(O2) Let $f, g \in \tau_k^N$. Since $N_k^x(f \odot g) \geq N_k^x(f) \odot N_k^x(g) = (f \odot g)_e(x)$ and (SN4), then $f \odot g \in \tau_k^N$.

(O3) Let $f_i \in \tau_k^N$ for all $i \in \Gamma$. Since $N_k^x(\bigsqcup_{i\in\Gamma} f_i) \geq \bigvee_{i\in\Gamma} N_k^x(f_i) = \bigsqcup_{i\in\Gamma}(f_i)_e(x)$ and (SN4), then $\bigsqcup_{i\in\Gamma} f_i \in \tau_k^N$.

(2) (R) Let $f \in \tau_k^N$. Since $N_k^x(\alpha \odot f) \geq \alpha \odot N_k^x(f) = \alpha \odot f_e(x)$ and (SN4), then $\alpha \odot f \in \tau_k^N$.

(3) Since $N_k^x(f) \leq N_k^x(N_k^-(f)) \leq N_k^x(f)$ from (SN3) and (SN5), $N_k^x(f) = N_k^x(N_k^-(f))$ for all $x \in X$. Since $N_k^-(f) \in \tau_k^N$, by the definition of $N_k^{\tau^N}$, $N_k^x(f) \leq (N^{\tau^N})_k^x(f)$.

Let $E = \{e\}$ be given. Since $N_x^{\tau^N}(f) = \bigvee\{(g_i)_e(x) \mid g_i \sqsubseteq f,\ g_i \in \tau^N\}$ and $(g_i)_e(x) = N_k^x(g_i)$, then

$$\bigvee_i (g_i)_e(x) = \bigvee_i N_k^x(g_i)_e(x) \leq N_k^x(N_-^{\tau^N}(f)) = N_k^x(\bigvee_i g_i) \leq \bigvee_i (g_i)_e(x).$$

Hence $N_k^x(N^{\tau^N})_k^-(f)) = (N^{\tau^N})_k^x(f)$. Since $(N^{\tau^N})_k^-(f) \sqsubseteq f$, by (SN3),

$$(N^{\tau^N})_k^x(f) = N_k^x((N^{\tau^N})_k^-(f)) \leq N_k^x(f).$$

Thus $(N^{\tau^N})_k^x = N_k^x$ for all $x \in X$.

(4) Let $f \in \tau_k^{N^\tau}$. Then $f = N_-^\tau(f) = \bigsqcup\{g \mid g \sqsubseteq f, g \in \tau_k\} \in \tau_k$. Hence $\tau^{N^\tau} \subset \tau_k$.

Let $g \in \tau_k$. Then $g_e(x) = (N^\tau)_k^x(g)$ for all $x \in X$. Then $g \in \tau_k^{N^\tau}$.

Theorem 12. *Let (X, τ_X) be a (K_1, E_1)-soft topological space and (Y, τ_Y) be a (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X, \tau_X) \to (Y, \tau_Y)$ is soft continuous, then $\varphi_{\psi,\eta} : (X, N^{\tau_X}) \to (Y, N^{\tau_Y})$ is soft N-continuous.*

Proof. Since $\varphi_\psi^{-1}(g) \in (\tau_X)_k$ for each $g \in (\tau_Y)_{\eta(k)}$, we have

$$\begin{aligned}&(N^{\tau_Y})^{\varphi(x)}_{\eta(k)}(f) = \bigvee\{g_{\psi(e)}(\varphi(x)) \mid g \sqsubseteq f, g \in (\tau_Y)_{\eta(k)}\}\\ &\leq \bigvee\{\varphi_\psi^{-1}(g)_e(x) \mid \varphi_\psi^{-1}(g) \sqsubseteq \varphi_\psi^{-1}(f), \varphi_\psi^{-1}(g) \in (\tau_X)_k\}\\ &\leq (N^{\tau_Y})^x_k(\varphi_\psi^{-1}(f)).\end{aligned}$$

Theorem 13. *Let (X, N_X) be a (K_1, E_1)-soft topological space and (Y, N_Y) be a (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X, N_X) \to (Y, N_Y)$ is soft N-continuous, then $\varphi_{\psi,\eta} : (X, \tau^{N_X}) \to (Y, \tau^{N_X})$ is soft continuous.*

Proof. Let $f \in (\tau_Y)_{\eta(k)}$. Since $\tau_Y = \tau^{N^{\tau_Y}}$ from Theorem 11(4), $f_{\psi(e)}(\varphi(x)) = (N^{\tau_Y})^{\varphi(x)}_{\eta(k)}(f) \leq (N^{\tau_Y})^x_k(\varphi_\psi^{-1}(f))$. Hence $\phi^{\leftarrow}(f) \in \tau_Y$.

Example 14. Let $X = \{h_i \mid i = \{1, 2, 3\}\}$ with h_i=house and $E = \{e, b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0, 1]$ by

$$x \odot y = \max\{0, x + y - 1\},\ x \to y = \min\{1 - x + y, 1\}$$

Then $([0, 1], \wedge, \to, 0, 1)$ is a stsc-quantle (ref [8,9]). Put $f, g \in (L^X)^E$ such that

$$\begin{aligned}&f_e(h_1) = 0.5, f_e(h_2) = 0.5, f_e(h_3) = 0.6\\ &f_b(h_1) = 0.6, f_b(h_2) = 0.3, f_b(h_3) = 0.6\\ &g_e(h_1) = 0.3, g_e(h_2) = 0.2, g_e(h_3) = 0.5\\ &g_b(h_1) = 0.4, g_b(h_2) = 0.4, g_b(h_3) = 0.1\end{aligned}$$

Put $K = \{k_1, K_2\}$. We define $\tau_{k_1}, \tau_{k_2} \subset [0, 1]^{([0,1]^X)^E}$ as follows:

$$\tau_{k_1} = \{1_X, 0_X, f, f \odot f\},\ \tau_{k_2} = \{1_X, 0_X, g\}.$$

We obtain a $[0, 1]$-fuzzy (K, E)-soft neighborhood system N^τ on X as:

$$(N^\tau)^{h_1}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X\\ 0.5, & \text{if } f \sqsubseteq h \neq 1_X,\\ 0, & \text{otherwise.}\end{cases} \qquad (N^\tau)^{h_1}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X\\ 0.3, & \text{if } g \sqsubseteq h \neq 1_X,\\ 0, & \text{otherwise.}\end{cases}$$

$$(N^\tau)^{h_2}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X\\ 0.3, & \text{if } f \sqsubseteq h \neq 1_X,\\ 0, & \text{otherwise.}\end{cases} \qquad (N^\tau)^{h_2}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X\\ 0.3, & \text{if } g \sqsubseteq h \neq 1_X,\\ 0, & \text{otherwise.}\end{cases}$$

$$(N^{\tau})^{h_3}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.6, & \text{if } f \sqsubseteq h \neq 1_X, \\ 0.2, & \text{if } f \odot f \sqsubseteq h \not\sqsupseteq f, \\ 0, & \text{otherwise.} \end{cases} \qquad (N^{\tau})^{h_3}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.1, & \text{if } g \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\tau^{N^{\tau}}_{k_1} = \{1_X, 0_X, f_1, f_1 \odot f_1\},\ \tau^{N^{\tau}}_{k_2} = \{1_X, 0_X, g_1\}.$$

$$((f_1)_e = (f_1)_b)(h_1) = 0.5, ((f_1)_e = (f_1)_b)(h_2) = 0.3,$$
$$((f_1)_e = (f_1)_b)(h_3) = 0.6, ((g_1)_e = (g_1)_b)(h_1) = 0.3,$$
$$((g_1)_e = (g_1)_b)(h_2) = 0.2, ((g_1)_e = (g_1)_b)(h_3) = 0.1.$$

We obtain a $[0,1]$-fuzzy (K,E)-soft neighborhood system $N^{\tau^{N^{\tau}}}$ on X as:

$$(N^{\tau^{N^{\tau}}})^{h_1}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.5, & \text{if } f_1 \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases} \qquad (N^{\tau^{N^{\tau}}})^{h_1}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.3, & \text{if } g_1 \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

$$(N^{\tau^{N^{\tau}}})^{h_2}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.3, & \text{if } f_1 \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases} \qquad (N^{\tau^{N^{\tau}}})^{h_2}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.3, & \text{if } g_1 \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

$$(N^{\tau^{N^{\tau}}})^{h_3}_{k_1}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.6, & \text{if } f_1 \sqsubseteq h \neq 1_X, \\ 0.2, & \text{if } f_1 \odot f_1 \sqsubseteq h \not\sqsupseteq f, \\ 0, & \text{otherwise.} \end{cases} \qquad (N^{\tau^{N^{\tau}}})^{h_3}_{k_2}(h) = \begin{cases} 1, & \text{if } h = 1_X \\ 0.1, & \text{if } g_1 \sqsubseteq h \neq 1_X, \\ 0, & \text{otherwise.} \end{cases}$$

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