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(K, E)-SOFT UNIFORMITIES AND L-FUZZY (K, E)-SOFT NEIGHBORHOOD SYSTEMS

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Abstract: In this paper, we define (K, E)-soft uniformities in complete residuated lattices. We investigate the relations among (K, E)-soft uniformities, (K, E)-soft topologies and L-fuzzy (K, E)-soft neighborhood systems. We give their examples.

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1. Introduction

Molodtsov [15,16] introduced the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. Maji et al. [12,13] gave the first practical application of soft sets in decision making problems. Many researchers have contributed towards the algebraic structure of soft set theory [1-5,7]. Shabir and Naz [23] introduced the study of soft

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topological spaces. They defined soft topology on the collection of soft sets over X and established their several properties. Aygünoglu et.al [2] introduced the concept of (K, E)-soft topology in the sense of Šostak [9]. Cetkin et.al [3] studied (K, E)-soft proximities and discuss their properties.

Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic and decision rules in complete residuated lattices. Höhle [9] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV-algebra). Ramadan et al. [10,11, 18,20] define the the concept of L- fuzzy (K, E)-soft topogenous orders, L-fuzzy (K, E)-soft uniform spaces, L-fuzzy (K, E)-soft topological spaces in strictly two sided commutative quantales and investigated the relation between them.

In this paper, we define (K, E)-soft uniformities in complete residuated lattices. We investigate the relations among (K, E)-soft uniformities, (K, E)-soft topologies and L-fuzzy (K, E)-soft neighborhood systems. We give their examples.

2. Preliminaries

Definition 1. [8] A structure $(L, \vee, \wedge, \odot, \rightarrow, 0, 1)$ is called a *complete residuated lattice* iff it satisfies the following properties:

(L1) $(L, \vee, \wedge, 0, 1)$ is a complete lattice where 0 is the bottom element and 1 is the top element;

(L2) $(L, \odot, 1)$ is a commutative monoid;

(L3) adjointness properties,i.e.

$$x \leq y \rightarrow z \text{ iff } x \odot y \leq z.$$

In this paper, we assume that $(L, \vee, \wedge, \odot, \rightarrow, ^*, 0, 1)$ be a complete residuated lattice.

Lemma 2. [8,9] *Let $(L, \vee, \wedge, \odot, \rightarrow, ^*, 0, 1)$ be a complete residuated lattice. For each $x, y, z, x_i, y_i \in L$, the following properties hold.*

(1) *If $y \leq z$, then $x \odot y \leq x \odot z$.*

(2) *If $y \leq z$, then $x \rightarrow y \leq x \rightarrow z$ and $z \rightarrow x \leq y \rightarrow x$.*

(3) *$x \rightarrow y = 1$ iff $x \leq y$.*

(4) *$x \rightarrow 1 = 1$ and $1 \rightarrow x = x$.*

(5) *$x \odot y \leq x \wedge y$.*

(6) *$x \odot (\bigvee_{i\in\Gamma} y_i) = \bigvee_{i\in\Gamma}(x \odot y_i)$ and $(\bigvee_{i\in\Gamma} x_i) \odot y = \bigvee_{i\in\Gamma}(x_i \odot y)$.*

(7) $x \to (\bigwedge_{i\in\Gamma} y_i) = \bigwedge_{i\in\Gamma}(x \to y_i)$ *and* $(\bigvee_{i\in\Gamma} x_i) \to y = \bigwedge_{i\in\Gamma}(x_i \to y)$.

(8) $\bigvee_{i\in\Gamma} x_i \to \bigvee_{i\in\Gamma} y_i \geq \bigwedge_{i\in\Gamma}(x_i \to y_i)$ *and* $\bigwedge_{i\in\Gamma} x_i \to \bigwedge_{i\in\Gamma} y_i \geq \bigwedge_{i\in\Gamma}(x_i \to y_i)$.

(9) $(x \to y) \odot x \leq y$ *and* $(y \to z) \odot (x \to y) \leq (x \to z)$.

(10) $x \to y \leq (y \to z) \to (x \to z)$ *and* $x \to y \leq (z \to x) \to (z \to y)$.

(11) $\bigwedge_{i\in\Gamma} x_i^* = (\bigvee_{i\in\Gamma} x_i)^*$ *and* $\bigvee_{i\in\Gamma} x_i^* = (\bigwedge_{i\in\Gamma} x_i)^*$.

(12) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$ *and* $(x \odot y)^* = x \to y^*$.

(13) $y \to z \leq x \odot y \to x \odot z$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 3. [4] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f_e^* = (f_e)^*$, for each $e \in E$.

(6) f is called a null L- fuzzy soft set and is denoted by 0_X , if $f_e(x) = 0$, for each $e \in E$, $x \in X$.

(7) f is called an absolute L- fuzzy soft set and is denoted by 1_X , if $f_e(x) = 1$, for each $e \in E$, $x \in X$ and $(1_X)_e(x) = 1$.

Definition 4. [4] Let $\varphi : X \to Y$ and $\psi : E_1 \to E_2$ be two mappings, where E_1 and E_2 are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ is called a fuzzy soft mapping.

(1) For $f \in (L^X)^{E_1}$, the image of f under the fuzzy soft mapping φ_ψ defined by, $\forall k \in K, \forall y \in Y$,

$$\varphi_\psi(f)_{e_2}(y) = \begin{cases} \bigvee_{\varphi(x)=y}(\bigvee_{\psi(e_1)=e_2} f_{e_1}(x)), & \text{if } x \in \varphi^{-1}(\{y\}), e_1 \in \psi^{-1}(\{e_2\}) \\ 0, & \text{otherwise.} \end{cases}$$

(2) For $g \in (L^X)^{E_2}$, the pre-image of g defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E_1, \forall x \in X.$$

(3) The soft mapping $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 5. [10] Let $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ be a soft mapping. Then we have the following properties. For $f, f_i \in (L^X)^{E_1}$ and $g, g_i \in (L^Y)^{E_2}$,

(1) $g \sqsupseteq \varphi_\psi(\varphi_\psi^{-1}(g))$ with equality if φ_ψ is surjective,

(2) $f \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(f))$ with equality if φ_ψ is injective,

(3) if φ_ψ is injective,

$$\varphi_\psi(f)_{e_2}(y) = \begin{cases} f_{e_1}(x), & \text{if } x \in \varphi^{-1}(y), e_1 \in \psi^{-1}(e_2) \\ 0, & \text{otherwise,} \end{cases}$$

(4) $\varphi_\psi^{-1}(g^*) = (\varphi_\psi^{-1}(g))^*$,

(5) $\varphi_\psi^{-1}(\bigvee_{i\in I} g_i) = \bigvee_{i\in I} \varphi_\psi^{-1}(g_i)$,

(6) $\varphi_\psi^{-1}(\bigwedge_{i\in I} g_i) = \bigwedge_{i\in I} \varphi_\psi^{-1}(g_i)$,

(7) $\varphi_\psi(\bigvee_{i\in I} f_i) = \bigvee_{i\in I} \varphi_\psi(f_i)$,

(8) $\varphi_\psi(\bigwedge_{i\in I} f_i) \sqsubseteq \bigwedge_{i\in I} \varphi_\psi(f_i)$ with equality if φ_ψ is injective,

(9) $\varphi_\psi^{-1}(g_1 \odot g_2) = \varphi_\psi^{-1}(g_1) \odot \varphi_\psi^{-1}(g_2)$,

(10) $\varphi_\psi(f_1 \odot f_2) \sqsubseteq \varphi_\psi(f_1) \odot \varphi_\psi(f_2)$ with equality if φ_ψ is injective,

Lemma 6. [20] Define a binary mapping $S : (L^X)^E \times (L^X)^E \to L$ by

$$S(f, g) = \bigwedge_{x\in X} \bigwedge_{e\in E} (f_e(x) \to g_e(x)) \;\; \forall\, f, g \in (L^X)^E, \;\; \forall\, e \in E.$$

Then $\forall f, g, h, m, n \in (L^X)^E$ the following statements hold.

(1) $f \sqsubseteq g$ iff $S(f, g) = 1$.

(2) If $f \sqsubseteq g$, then $S(h, f) \le S(h, g)$ and $S(f, h) \ge S(g, h)$.

(3) $S(f, h) \odot S(h, g) \le S(f, g)$. Moreover, $\bigvee_{h\in(L^X)^E}(S(f, h) \odot S(h, g)) = S(f, g)$

(4) $S(f, g) \odot S(m, n) \le S(f \odot m, g \odot n)$.

(5) If $\varphi_\psi : (X, E) \to (Y, F)$ is a fuzzy soft mapping, then $S(p, q) \le S(\varphi_\psi^{-1}(p), \varphi_\psi^{-1}(q))$, for each $p, q \in (L^Y)^F$.

Definition 7. [11] A set $\tau = \{\tau_k \subset P((L^X)^E) \mid k \in K\}$ for each $k \in K$ is called a (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(SO1) $0_X, 1_X \in \tau_k$,

(SO2) If $f, g \in \tau_k$, then $(f \odot g) \in \tau_k$.

(SO3) If $f_i \in \tau_k$, $\bigsqcup_{i\in I} f_i \in \tau_k$.

The pair (X, τ) is called a (K, E)-soft topological space. Let (X, τ^1) be a (K_1, E_1)-soft topological space and (Y, τ^2) be a (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X, τ^1) into (Y, τ^2) is called soft continuous if $\varphi_\psi^{-1}(f) \in (\tau^1)_k \;\; \forall\, f \in (\tau^2)_{\eta(k)}, k \in K_1$.

Definition 8. A set $U = \{U_k \subset P((L^{X\times X})^E) \mid k \in K\}$ is called a (K, E)-soft quasi-uniformity on X iff the following conditions are fulfilled

(QU1) $1_{X\times X} \in U_k$,

(QU2) If $v \leq u$ and $v \in U_k$, then $u \in U_k$,

(QU3) For every $u, v \in U_k$, $u \odot v \in U_k$,

(QU4) If $u \in U_k$ then $1_\triangle \leq u$ where

$$(1_\triangle)_e(x, y) = \begin{cases} 1, & \text{if x = y} \\ 0, & \text{if } x \neq y, \end{cases}$$

(QU5) For each $u \in U_k$, there exists $v \in U_k$ such that $v \circ v \leq u$ where

$$(v \circ v)_e(x, y) = \bigvee_{z\in X} v_e(x, z) \odot v_e(z, y), \quad \forall\, x, y \in X, e \in E.$$

The pair (X, U) is called a (K, E)-soft quasi-uniform space.

A (K, E)-soft quasi-uniformity U on X is said to be stratified if

(S) if $u \in U_k$, then $\alpha \odot u \in U_k$.

A (K, E)-soft quasi-uniformity U on X is said to be (K, E)-soft uniformity if

(U) if $u \in U_k$, then $u^{-1} \in U_k$ where $(u^{-1})_e(x, y) = u_e(y, x)$.

Let (X, U^1) be a (K_1, E_1)-soft quasi-uniform space and (Y, U^2) be a (K_2, E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X, U^1) into (Y, U^2) is called soft uniformly continuous if $(\varphi \times \varphi)_\psi^{-1}(v) \in (U^1)_k \;\; \forall\, v \in (U^2)_{\eta(k)}, k \in K_1$.

Definition 9. [20] An L-fuzzy (K, E)-soft neighborhood system on X is a set $N = \{N^x \mid x \in X\}$ of mappings $N^x : K \to L^{(L^X)^E}$ such that for each $k \in K$:

(SN1) $N^x_k(1_X) = 1$ and $N^x_k(0_X) = 0$,

(SN2) $N^x_k(f \odot g) \geq N^x_k(f) \odot N_x(g)$ for each $f, g \in (L^X)^E$,

(SN3) If $f \sqsubseteq g$, then $N^x_k(f) \leq N^x_k(g)$,

(SN4) $N^x_k(f) \leq f_e(x)$ for all $f \in (L^X)^E$ and $e \in E$.

(SN5) $N^x_k(f) \leq \bigvee\{N^x_k(g) \mid g_e(y) \sqsubseteq N^y_k(f), \; \forall\, y \in X, e \in E\}$.

The previous axiom can be reformulated in the following way

(SN5) $\forall\ f \in (L^X)^E$ and $x \in X$, $N_k^x(f) \leq N_k^x(N_k^-(f))$, where $N_k^-(f) \in (L^X)^E$ is defined by

$$(N_k^-(f))_e(y) = N_k^y(f) \ \ \forall\ y \in Y, e \in E.$$

An L-fuzzy (K,E)-soft neighborhood system is called stratified if

(SR) $N_k^x(\alpha \odot f) \geq \alpha \odot N_k^x(f)$ for all $f \in (L^X)^E$ and $\alpha \in L$.

The pair (X,N) is called an L-fuzzy (K,E)-soft neighborhood space.

Let (X,N) be an L-fuzzy (K_1,E_1)-soft neighborhood space and (Y,M) be an L-fuzzy (K_2,E_2)-soft neighborhood space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X,N) into (Y,M) is called soft N-continuous at every $x \in X$ if $M_{\eta(k)}^{\phi(x)}(f) \leq N_k^x(\varphi_\psi^{-1}(f)) \ \forall\ f \in (L^Y)^{E_2}, k \in K_1$.

Theorem 10. *Let (X,τ) be a (K,E)-soft topological space. Define a map $N_k^\tau : X \to L^{(L^X)^E}$ by*

$$(N^\tau)_k^x(f) = \bigvee \{ \bigwedge_{e \in E} g_e(x) \mid g \sqsubseteq f,\ g \in \tau_k \}.$$

Then the following properties hold.

(1) (X,N^τ) is a L-fuzzy (K,E)-soft neighborhood space.

(2) If τ is enriched, then N^τ is stratified and

$$(N^\tau)_k^x(f) = \bigvee_{g \in \tau} (\bigwedge_{e \in E} g_e(x) \odot S(g,f).$$

3. (K,E)-Soft Uniformities and L-fuzzy (K,E)-Soft Neighborhood Systems

Theorem 11. *Let (X,U) be an (K,E)-soft quasi uniform space. Define two maps $(rN^U)_k^-, (rN^U)_k^- : X \to L^{(L^X)^E}$ by*

$$(rN^U)_k^x(f) = \bigvee_{u \in U_k} S(u[x],f), \ \ \forall\ f \in (L^X)^E,\ x \in X,$$

$$(lN^U)_k^x(f) = \bigvee_{u \in U_k} S(u[[x]],f), \ \ \forall\ \ f \in (L^X)^E,\ x \in X,$$

where $(u[x])_e(y) = u_e(y,x)$ and $(u[[x]])_e(y) = u_e(x,y)$.

Then

(1) (X, rN^U) *is a stratified* L*-fuzzy* (K,E)*-soft neighborhood space.*

(2) (X, lN^U) *is a stratified* L*-fuzzy* (K,E)*-soft neighborhood space.*

(3) $(rN^U)^x_k(f) = \bigvee \{\bigwedge_{e\in X} g_e(x) \mid u[g] \sqsubseteq f, u \in U_k\} = \bigvee\{\bigwedge_{e\in X} g_e(x) \odot S(u[g], f) \mid u \in U_k\}$ *where*

$$(u[g])_e(x) = u_e[g_e](x) = \bigvee_{y\in X} u_e(x,y) \odot g_e(y),$$

(4) $(lN^U)^x_k(f) = \bigvee \{\ g(x) \mid u[[g]] \sqsubseteq f \mid u \in U\ \} = \bigvee\{\bigwedge_{e\in X} g_e(x) \odot S(u[[g]], f) \mid u \in U\}$ *where*

$$(u[[g]])_e(x) = u_e[[g_e]](x) = \bigvee_{y\in X} u_e(y,x) \odot g_e(y),$$

Proof. (1) (SN1) For $u \in U$, by (QU4), $1_\triangle \sqsubseteq u$. Then

$$\begin{array}{rl}(rN^U)^x_k(0_X) & = \bigvee_{u\in U_k} S(u[x], 0_X) \\ & \le \bigvee_{u\in U_k} (u_e(x,x) \to 0) = 0.\end{array}$$

Hence $(rN^U)^x_k(0_X) = 0$. Also, $(rN^U)^x_k(1_X) = 1$, because

$$(rN^U)^x_k(1_X) \ge \bigwedge_{y\in X} ((1_\triangle)_e(x,y) \to (1_X)_e(y)) = 1.$$

(SN2) By Lemma 6(4) , we have

$$\begin{array}{l}(rN^U)^x_k(f) \odot (rN^U)^x_k(g) = \left(\bigvee_{u\in U_k} S(u[x], f)\right) \odot \left(\bigvee_{v\in U_k} S(v[x], g)\right) \\ = \bigvee_{u\odot v\in U_k} S(u[x], f) \odot S(v[x], g) \le \bigvee_{u\odot v\in U_k} S((u\odot v)[x], f \odot g) \\ \le \bigvee_{w\in U_k} S(w[x], f \odot g) = (rN^U)^x_k(f \odot g).\end{array}$$

(SN3) By Lemma 6(3), we have

$$\begin{array}{rl}(rN^U)^x_k(f) & = \bigvee_{u\in U_k} S(u[x], f) \\ & \le \bigvee_{u\in U_k} S(u[x], g) = (rN^U)^x_k(g).\end{array}$$

(SN4) For $u \in U$, by (QU4), $1_\triangle \sqsubseteq u$. We have

$$\begin{array}{rl}(rN^{U_k})^x_k(f) & = \bigvee_{u\in U_k} \bigwedge_{y\in X} \bigwedge_{e\in E} (u_e(y,x) \to f_e(y)) \\ & \le \bigvee_{u\in U_k} (u_e(x,x) \to f_e(x)) \le f_e(x).\end{array}$$

(SN5)

$$\begin{array}{l}(rN^U)^x_k(f)=\bigvee_{u\in U_k}S(u[x],f)\\ =\bigvee_{u\in U_k}\bigwedge_{y\in X}\bigwedge_{e\in E}(u_e(y,x)\to f_e(y))\\ \le\bigvee_{v\in U_k}\bigwedge_{y\in X}\bigwedge_{e\in E}((v_e\circ v_e)(y,x)\to f_e(y))\\ =\bigvee_{v\in U_k}\bigwedge_{y\in X}\bigwedge_{e\in E}((\bigvee_{z\in X}v_e(z,x)\odot v_e(y,z))\to f_e(y))\\ =\bigvee_{v\in U_k}\bigwedge_{y\in X}\bigwedge_{e\in E}\bigwedge_{z\in X}((v_e(z,x)\odot v_e(y,z))\to f_e(y))\\ \text{(by Lemma 2 (12))}\\ =\bigvee_{v\in U_k}\bigwedge_{y\in X}\bigwedge_{e\in E}\bigwedge_{z\in X}(v_e(z,x)\to(v_e(y,z)\to f_e(y))\\ =\bigvee_{v\in U_k}\bigwedge_{z\in X}(v_e(z,x)\to\bigwedge_{y\in X}(v_e(y,z)\to f_e(y)).\end{array}$$

Put $g_e(z)=\bigwedge_{y\in X}(v_e(y,z)\to f_e(y))$. For all $g_e(z)\le(rN^U)^z_k(f)$ for each $z\in X,e\in E$, $\bigwedge_{e\in E}g_e(x)\le(rN^U)^z_k(f)$. Thus,

$$\begin{array}{l}(rN^U)^x_k(f)\\ \le\bigvee_{v\in U_k}\{\bigwedge_{z\in X}\bigwedge_{e\in E}(v_e(z,x)\to g_e(z))\mid g_e(z)\le(rN^U)^z_k(f)\}\\ \le\bigvee_v\left\{(rN^U)^x_k(g)\mid g_e(z)\le(rN^U)^z_k(f)\right\}.\end{array}$$

Thus, (X,rN^U) is an L-fuzzy (K,E)-soft neighborhood space.

Since $\alpha\odot(u[x])_e(y)\odot S(u[x],f)\le\alpha\odot(u[x])_e(y)\odot((u[x])_e(y)\to f_e(y))\le\alpha\odot f_e(y)$, we have

$$\alpha\odot S(u[x],f)\le S(u[x],\alpha\odot f).$$

Thus, rN^U is stratified from:

$$\begin{array}{rl}\alpha\odot(rN^U)^x_k(f)&=\alpha\odot\bigvee_{u\in U_k}S(u[x],f)=\bigvee_{u\in U_k}(\alpha\odot S(u[x],f))\\ &\le\bigvee_{u\in U_k}(S(u[x],\alpha\odot f))=(rN^U)^x_k(\alpha\odot f).\end{array}$$

(2) It is similarly proved as (1).

(3) Put $\gamma=\bigvee\{\bigwedge_{e\in X}g_e(x)\mid u[g]\sqsubseteq f,u\in U_k\}$. We show that $(rN^U)^x_k(f)=\gamma$ from the following statements.

Let $g_e(y)=\bigwedge_{x\in X}(u_e(x,y)\to f_e(x))$. Then

$$\begin{array}{l}u_e[g_e](z)=\bigvee_{y\in X}(u_e(z,y)\odot g_e(y))\\ =\bigvee_{y\in X}(u_e(z,y)\odot(\bigwedge_{x\in X}(u_e(x,y)\to f_e(x))))\\ \le\bigvee_{y\in X}(u_e(z,y)\odot(u_e(z,y)\to f_e(z)))\le f_e(z).\end{array}$$

Hence $(rN^U)^x_k(f)\le\gamma$.

Let $u_e[g_e](z)=\bigvee_{y\in X}(u_e(z,y)\odot g_e(y))\le f_e(z)$. Then

$$g_e(y)\le\bigwedge_{z\in X}(u_e(z,y)\to f_e(z)).$$

Hence $(rN^U)^x_k(f) \geq \gamma$.

Put $\delta = \bigvee\{(\bigwedge_{e\in E} g_e(x)) \odot S(u[g], f) \mid u \in U_k\}$. We show that $\delta = \gamma$ from the following statements.

Let $g \in (L^X)^E$ with $u[g] \leq f$ and $u \in U_k$. Then $S(u[g], f) = 1$. Hence $(\bigwedge_{e\in E} g_e(x)) \odot S(u[g], f) = (\bigwedge_{e\in E} g_e(x)) \leq \delta(x)$. So, $\gamma \leq \delta$.

Let $(\bigwedge_{e\in E} g_e) \odot S(u[g], f)$ with $u \in U$. Since

$$u_e[(\bigwedge_{e\in E} g_e) \odot S(u[g], f)](x) = \bigvee_{y\in X}((u_e(x,y) \odot (\bigwedge_{e\in E} g_e(y))) \odot S(u[g], f))$$
$$\leq u_e[g_e](x) \odot S(u[g], f) \leq f_e(x).$$

we have $u_e[(\bigwedge_{e\in E} g_e) \odot S(u[g], f)] \leq f_e$. Then $\bigwedge_{e\in E} g_e(x) \odot S(u[g], f) \leq \gamma$. Thus, $\delta = \gamma$.

Theorem 12. *Let (X, U) be a (K,E)-soft quasi-uniform space, (X, rN^U) and (X, lN^U) L-fuzzy (K,E)-soft neighborhood spaces. Define $(\tau^r_U)_k, (\tau^l_U)_k \subset (L^X)^E$ as follows*

$$(\tau^r_U)_k = \{f \in (L^X)^E \mid f_e(x) = (rN^U)^x_k(f), \forall x \in X, e \in E\},$$
$$(\tau^l_U)_k = \{f \in (L^X)^E \mid f_e(x) = (lN^U)^x_k(f), \forall x \in X, e \in E\}.$$

Then,
(1) $\tau^r_U = \{(\tau^r_U)_k \mid k \in K\}$ is an enriched (K,E)-soft topology on X.
(2) $\tau^l_U = \{(\tau^l_U)_k \mid k \in K\}$ is an enriched (K,E)-soft topology on X.
(3) $rN^U = N^{\tau^r_U}$.
(4) $lN^U = N^{\tau^l_U}$.

Proof. (1) (SO1) Since $(rN^U)^x_k(1_X) = 1$ and $(rN^U)^x_k(0_X) = 0$, we have $1_X, 0_X \in (\tau^r_U)_k$.

(SO2) Let $f, g \in (\tau^r_U)_k$ with $(rN^U)^x_k(f) = f_e(x)$ and $(rN^U)^x_k(g) = g_e(x)$. Since $(rN^U)^x_k(f \odot g) \geq (rN^U)^x_k(f) \odot (rN^U)^x_k(g) = (f \odot g)_e(x)$ and (SN4), then $f \odot g \in (\tau^r_U)_k$.

(SO3) Let $f_i \in (\tau^r_U)_k$ for all $i \in \Gamma$. Since $(rN^U)^x_k(\bigsqcup_{i\in\Gamma} f_i) \geq \bigvee_{i\in\Gamma}(rN^U)^x_k(f_i) = \bigsqcup_{i\in\Gamma}(f_i)_e(x)$ and (SN4), then $\bigsqcup_{i\in\Gamma} f_i \in (\tau^r_U)_k$.

(R) Let $f \in (\tau^r_U)_k$. Since $(rN^U)^x_k(\alpha \odot f) \geq \alpha \odot (rN^U)^x_k(f) = \alpha \odot f_e(x)$ and (SN4), then $\alpha \odot f \in (\tau^r_U)_k$.

(2) It is similarly proved as (1).

(3) Since $(rN^U)^x_k(f) \leq (rN^U)^x_k((rN^U)^-_k(f)) \leq (rN^U)^x_k(f)$ from (SN3) and (SN5), $(rN^U)^x_k(f) = (rN^U)^x_k((rN^U)^-_k(f))$ for all $x \in X$. Since $(rN^U)^-_k(f) \in \tau^r_U$, by the definition of $N^{\tau^r_U}$, $(rN^U)^x_k(f) \leq (N^{\tau^r_U})^x_k(f)$.

Since $(N^{\tau_U^r})_k^x(f) = \bigvee\{\bigwedge_{e\in E}(g_i)_e(x) \mid g_i \sqsubseteq f,\ g_i \in (\tau_U^r)_k\}$ and $\bigwedge_{e\in E}(g_i)_e(x) = (g_i)_e(x) = (rN^U)_k^x(g_i)$, then

$$\begin{aligned}&\bigvee_i (g_i)_e(x) = \bigvee_i (rN^U)_k^x(g_i) \leq (rN^U)_k^x((N^{\tau_U^r})_k^-(f))\\ &= (rN^U)_k^x(\bigsqcup_i g_i) \leq \bigvee_i (g_i)_e(x).\end{aligned}$$

Hence $(rN^U)_k^x((N^{\tau_U^r})_k^-(f)) = (N^{\tau_U^r})_k^x(f)$. Since $(N^{\tau_U^r})_k^x(f) \leq f_e(x)$ for all $e \in E$, by (SN3), $(N^{\tau_U^r})_k^x(f) = (rN^U)_k^x(N_k^{\tau_U^r}(f)) \leq (rN^U)_k^x(f)$. So, $rN^U = N^{\tau_U^r}$.

(4) It is similarly proved as (3).

Theorem 13. *Let (X,U) be a (K_1,E_1)-soft quasi-uniform space and (Y,U) be a (K_2,E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X,U) \to (Y,V)$ is soft uniformly continuous, then*

(1) $\varphi_{\psi,\eta} : (X, rN^U) \to (Y, rN^V)$ *is soft N-continuous.*

(2) $\varphi_{\psi,\eta} : (X, lN^U) \to (Y, lN^V)$ *is soft N-continuous.*

(3) $\varphi_{\psi,\eta} : (X, \tau_U^r) \to (Y, \tau_V^r)$ *is soft continuous.*

(4) $\varphi_{\psi,\eta} : (X, \tau_U^l) \to (Y, \tau_V^l)$ *is soft continuous.*

Proof. (1) First we show that $\varphi_\psi^{-1}((v[\varphi(x)])_{\psi(e)}) = ((\varphi\times\varphi)_\psi^{-1}(v)[x])_e$ from

$$\begin{aligned}&\varphi_\psi^{-1}((v[\varphi(x)])_{\psi(e)})(z) = (v[\varphi(x)])_{\psi(e)}(\varphi(z)) = v_{\psi(e)}(\varphi(z),\varphi(x))\\ &= (\varphi\times\varphi)_\psi^{-1}(v_{\psi(e)})(z,x) = ((\varphi\times\varphi)_\psi^{-1}(v)[x])_e(z).\end{aligned}$$

Thus we have

$$\begin{aligned}S(v[\varphi(x)],f) &= \bigwedge_{y\in Y}\bigwedge_{e_2\in E_2}(v[\varphi(x)]_{e_2}(y) \to f_{e_2}(y))\\ &\leq \bigwedge_{z\in X}\bigwedge_{e_1\in E_1}(v[\varphi(x)]_{\psi(e_1)}(\varphi(z)) \to f_{\psi(e_1)}(\varphi(z)))\\ &= \bigwedge_{z\in X}\bigwedge_{e_1\in E_1}(\varphi_\psi^{-1}((v[\varphi(x)])_{\psi(e_1)})(z) \to \varphi_\psi^{-1}(f)_{e_1}(z))\\ &= \bigwedge_{z\in X}\bigwedge_{e_1\in E_1}(((\varphi\times\varphi)_\psi^{-1}(v)[x])_{e_1}(z) \to \varphi_\psi^{-1}(f)_{e_1}(z))\\ &= S((\varphi\times\varphi)_\psi^{-1}(v)[x], \varphi_\psi^{-1}(f)).\end{aligned}$$

$$\begin{aligned}&(rN^V)_{\eta(k)}^{\varphi(x)}(f) = \bigvee_{v\in V_{\eta(k)}} S(v[\phi(x)],f)\\ &\leq \bigvee_{v\in V_{\eta(k)}} S((\varphi\times\varphi)_\psi^{-1}(v)[x], \varphi_\psi^{-1}(f))\\ &\leq \bigvee_{(\varphi\times\varphi)_\psi^{-1}(v)\in U_k} S((\varphi\times\varphi)_\psi^{-1}(v)[x], \varphi_\psi^{-1}(f)) \leq (rN^U)_k^x(\varphi_\psi^{-1}(f)).\end{aligned}$$

(2) It is similarly proved as (1).

(3) Let $f \in (\tau_V^r)_{\eta(k)}(f)$. Then $f_{\psi(e)}(\varphi(x)) = (rN^V)_{\eta(k)}^{\varphi(x)}(f)$. Then $\varphi_\psi^{-1}(f)_e(x) = \varphi_\psi^{-1}((rN^V)_{\eta(k)}^-(f))_e(x)$. Since $(rN^V)_{\eta(k)}^{\varphi(x)}(f) \leq (rN^U)_k^x(\varphi_\psi^{-1}(f))$,

$$\varphi_\psi^{-1}(f)_e(x) = \varphi_\psi^{-1}((rN^V)_{\eta(k)}^-(f))_e(x) = (rN^V)_{\eta(k)}^{\varphi(x)}(f) \leq (rN^U)_k^x(\varphi_\psi^{-1}(f)).$$

By (SN3), $\varphi_\psi^{-1}(f) = (rN^U)_k^-(\varphi_\psi^{-1}(f))$. Hence $\varphi_\psi^{-1}(f) \in (\tau_U^r)_k$.

(4) It is similarly proved as (3).

Example 14. Let $X = \{h_i \mid i = \{1, 2, 3\}\}$ with h_i=house and $E = \{e, b\}$ with e=expensive, b= beautiful. Let $(L = [0, 1], \odot = \wedge, \to, 0, 1)$ be a complete residuated lattice defined by

$$x \odot y = x \wedge y, \quad x \to y = \begin{cases} 1, & \text{if } x \leq y, \\ y, & \text{otherwise.} \end{cases}$$

Put $f \in (L^X)^E$ such that

$$\begin{array}{l} f_e(h_1) = 0.3, f_e(h_2) = 0.5, f_e(h_3) = 0.3 \\ f_b(h_1) = 0.7, f_b(h_2) = 0.9, f_b(h_3) = 0.4 \end{array}$$

Put $K = \{k_1, K_2\}$ and $w, v \in (L^{X \times X})^E$ such that

$$w_e = \begin{pmatrix} 1 & 0.3 & 0.5 \\ 0.6 & 1 & 0.7 \\ 0.5 & 0.3 & 1 \end{pmatrix}, w_b = \begin{pmatrix} 1 & 0.5 & 0.4 \\ 0.5 & 1 & 0.4 \\ 0.4 & 0.4 & 1 \end{pmatrix},$$

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.8 \\ 0.4 & 1 & 0.4 \\ 0.5 & 0.5 & 1 \end{pmatrix}, v_b = \begin{pmatrix} 1 & 0.5 & 0.4 \\ 0.7 & 1 & 0.4 \\ 0.6 & 0.5 & 1 \end{pmatrix}.$$

Define $U_{k_1} = \{u \in (L^{X \times X})^E \mid u \geq w\}$ and $U_{k_2} = \{u \in (L^{X \times X})^E \mid u \geq v\}$.

(1) Since $w_e \circ w_e = w_e$ and $v_e \circ v_e = v_e$ for all $e \in E$, $U = \{U_{k_1}, U_{k_2}\}$ is a (K, E)-soft quasi-uniformity on X.

(2) Since $(rN^U)_k^x(f) = \bigvee_{u \in U_k} S(u[x], f)$, we have

$$\begin{array}{l} (rN^U)_{k_1}^{h_1}(f) = \bigvee_{u \in U_{k_1}} S(u[h_1], f) = f_e(h_1) \wedge (0.6 \to f_e(h_2)) \\ \wedge (0.5 \to f_e(h_3)) \wedge f_b(h_1) \wedge (0.5 \to f_b(h_2)) \wedge (0.4 \to f_b(h_3)), \\ (rN^U)_{k_1}^{h_2}(f) = \bigvee_{u \in U_{k_1}} S(u[h_2], f) = (0.3 \to f_e(h_1)) \wedge f_e(h_2) \\ \wedge (0.3 \to f_e(h_3)) \wedge (0.5 \to f_b(h_1)) \wedge f_b(h_2) \wedge (0.4 \to f_b(h_3)), \\ (rN^U)_{k_1}^{h_3}(f) = \bigvee_{u \in U_{k_1}} S(u[h_3], f) = (0.5 \to f_e(h_1)) \wedge (0.7 \to f_e(h_2)) \\ \wedge f_e(h_3) \wedge (0.4 \to f_b(h_1)) \wedge (0.4 \to f_b(h_2)) \wedge f_b(h_3). \end{array}$$

Then $(rN^U)^{h_1}_{k_1}(f)=0.3, (rN^U)^{h_2}_{k_1}(f)=0.5, (rN^U)^{h_3}_{k_1}(f)=0.3.$

$$\begin{array}{l}(rN^U)^{h_1}_{k_2}(f)=\bigvee_{u\in U_{k_2}} S(u[h_1],f)=f_e(h_1)\wedge(0.4\to f_e(h_2))\\ \wedge(0.5\to f_e(h_3))\wedge f_b(h_1)\wedge(0.7\to f_b(h_2))\wedge(0.6\to f_b(h_3)),\\ (rN^U)^{h_2}_{k_2}(f)=\bigvee_{u\in U_{k_2}} S(u[h_2],f)=(0.6\to f_e(h_1))\wedge f_e(h_2)\\ \wedge(0.5\to f_e(h_3))\wedge(0.5\to f_b(h_1))\wedge f_b(h_2)\wedge(0.5\to f_b(h_3)),\\ (rN^U)^{h_3}_{k_2}(f)=\bigvee_{u\in U_{k_2}} S(u[h_3],f)=(0.8\to f_e(h_1))\wedge(0.4\to f_e(h_2))\\ \wedge f_e(h_3)\wedge(0.4\to f_b(h_1))\wedge(0.4\to f_b(h_2))\wedge f_b(h_3).\end{array}$$

Then $(rN^U)^{h_1}_{k_2}(f)=0.3, (rN^U)^{h_2}_{k_2}(f)=0.3, (rN^U)^{h_3}_{k_2}(f)=0.3.$

(3) Since $(lN^U)^x_k(f)=\bigvee_{u\in U} S(u[[x]],f)$, we have

$$\begin{array}{l}(lN^U)^{h_1}_{k_1}(f)=\bigvee_{u\in U_{k_1}} S(u[[h_1]],f)=f_e(h_1)\wedge(0.3\to f_e(h_2))\\ \wedge(0.5\to f_e(h_3))\wedge f_b(h_1)\wedge(0.5\to f_b(h_2))\wedge(0.4\to f_b(h_3)),\\ (lN^U)^{h_2}_{k_1}(f)=\bigvee_{u\in U_{k_1}} S(u[h_2],f)=(0.6\to f_e(h_1))\wedge f_e(h_2)\\ \wedge(0.7\to f_e(h_3))\wedge(0.5\to f_b(h_1))\wedge f_b(h_2)\wedge(0.4\to f_b(h_3)),\\ (lN^U)^{h_3}_{k_1}(f)=\bigvee_{u\in U_{k_1}} S(u[h_3],f)=(0.5\to f_e(h_1))\wedge(0.3\to f_e(h_2))\\ \wedge f_e(h_3)\wedge(0.4\to f_b(h_1))\wedge(0.4\to f_b(h_2))\wedge f_b(h_3).\end{array}$$

Then $(lN^U)^{h_1}_{k_1}(f)=0.3, (lN^U)^{h_2}_{k_1}(f)=0.3, (lN^U)^{h_3}_{k_1}(f)=0.3.$

$$\begin{array}{l}(lN^U)^{h_1}_{k_2}(f)=\bigvee_{u\in U_{k_2}} S(u[[h_1]],f)=f_e(h_1)\wedge(0.6\to f_e(h_2))\\ \wedge(0.8\to f_e(h_3))\wedge f_b(h_2)\wedge(0.5\to f_b(h_2))\wedge(0.4\to f_b(h_3)),\\ (lN^U)^{h_2}_{k_2}(f)=\bigvee_{u\in U_{k_2}} S(u[h_2],f)=(0.4\to f_e(h_1))\wedge f_e(h_2)\\ \wedge(0.4\to f_e(h_3))\wedge(0.7\to f_b(h_1))\wedge f_b(h_2)\wedge(0.4\to f_b(h_3)),\\ (lN^U)^{h_3}_{k_2}(f)=\bigvee_{u\in U_{k_2}} S(u[h_3],f)=(0.5\to f_e(h_1))\wedge(0.5\to f_e(h_2))\\ \wedge f_e(h_3)\wedge(0.6\to f_b(h_1))\wedge(0.5\to f_b(h_2))\wedge f_b(h_3).\end{array}$$

Then $(lN^U)^{h_1}_{k_1}(f)=0.3, (lN^U)^{h_2}_{k_1}(f)=0.3, (lN^U)^{h_3}_{k_1}(f)=0.3.$

(4) Since $(\tau^r_U)_k=\{f\in(L^X)^E \mid f_e(x)=(rN^U)^x_k(f), \forall x\in X, e\in E\}$ from Theorem 12, $f_e=f_b$, we have

$$f\in(\tau^r_U)_{k_1} \text{ iff } \begin{cases} f=\alpha_X,\\ f_e(h_1)\le 0.6\to f_e(h_2), f_e(h_1)\le 0.5\to f_e(h_3),\\ f_e(h_2)\le 0.5\to f_e(h_1), f_e(h_2)\le 0.4\to f_e(h_3),\\ f_e(h_3)\le 0.5\to f_e(h_1), f_e(h_3)\le 0.7\to f_e(h_3),\end{cases}$$

$$f \in (\tau_U^r)_{k_1} \text{ iff } \begin{cases} f = \alpha_X, \\ f_e(h_1) \leq 0.7 \to f_e(h_2), f_e(h_1) \leq 0.6 \to f_e(h_3), \\ f_e(h_2) \leq 0.6 \to f_e(h_1), f_e(h_2) \leq 0.5 \to f_e(h_3), \\ f_e(h_3) \leq 0.8 \to f_e(h_1), f_e(h_3) \leq 0.4 \to f_e(h_3), \end{cases}$$

Put $(g_e = g_b)(h_1) = 0.5, (g_e = g_b)(h_2) = 0.8, (g_e = g_b)(h_3) = 0.6$. Then $g \in (\tau_U^r)_{k_1}$ but $g \notin (\tau_U^r)_{k_2}$ because

$$f_e(h_3) = 0.6 \not\leq 0.8 \to f_e(h_3) = 0.5.$$

(5) Since $(\tau_U^l)_k = \{f \in (L^X)^E \mid f_e(x) = (lN^U)_k^x(f), \forall x \in X, e \in E\}$ from Theorem 12, $f_e = f_b$, we have

$$f \in (\tau_U^l)_{k_1} \text{ iff } \begin{cases} f = \alpha_X, \\ f_e(h_1) \leq 0.5 \to f_e(h_2), f_e(h_1) \leq 0.5 \to f_e(h_3), \\ f_e(h_2) \leq 0.6 \to f_e(h_1), f_e(h_2) \leq 0.7 \to f_e(h_3), \\ f_e(h_3) \leq 0.5 \to f_e(h_1), f_e(h_3) \leq 0.4 \to f_e(h_3), \end{cases}$$

$$f \in (\tau_U^l)_{k_2} \text{ iff } \begin{cases} f = \alpha_X, \\ f_e(h_1) \leq 0.6 \to f_e(h_2), f_e(h_1) \leq 0.8 \to f_e(h_3), \\ f_e(h_2) \leq 0.7 \to f_e(h_1), f_e(h_2) \leq 0.4 \to f_e(h_3), \\ f_e(h_3) \leq 0.6 \to f_e(h_1), f_e(h_3) \leq 0.5 \to f_e(h_3), \end{cases}$$

Put $(h_e = h_b)(h_1) = 0.8, (h_e = h_b)(h_2) = 0.5, (h_e = h_b)(h_3) = 0.9$. Then $h \in (\tau_U^l)_{k_1}$ but $h \notin (\tau_U^l)_{k_2}$ because

$$f_e(h_1) = 0.8 \not\leq 0.6 \to f_e(h_2) = 0.5.$$

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