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CUT SETS ON TRAPEZOIDAL FUZZY NUMBER AND INTUITIONISTIC FUZZY NUMBER: A NEW PERSPECTIVE

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Abstract: The human cognition and interaction with the outside world involves structure with no sharp boundaries in which the transition of membership to non-membership function is gradual rather than abrupt. The concept of cut sets and fuzzy numbers were developed and intensive research has been taking place and applied in human cognition. In this paper, we have introduced matrix-cut for trapezoidal fuzzy number by studying its properties, arithmetic operations and decomposition theorems. Finally, the Trapezoidal Intuitionistic Fuzzy number (TFN) and its arithmetic properties using matrix cut have also been proposed.

AMS Subject Classification: 03E72
Key Words: fuzzy sets, trapezoidal fuzzy numbers, decomposition theorems, intuitionistic fuzzy sets, trapezoidal intuitionistic fuzzy numbers

1. Introduction

The theory of fuzzy sets developed by Zadeh in 1965 is a step towards rapprochement between the classical mathematics' precise image and the ever pervasive imprecision in the real world. It is a peacemaker born for better understanding of mental processes and cognition. The cells are the building blocks of living

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beings, likewise numbers and sets form the foundation in mathematics. The classical set theory is a particular case of fuzzy subset theory, with the reference set not being a fuzzy set. The membership of an element in fuzzy set theory is a value between 0 and 1. But in reality it is not always true that $\upsilon_A(x) = 1 - \mu_A(x)$. After the emergence of the concept of fuzzy sets, the sets were generalized in [8], who paved the way for L-Fuzzy sets. In May 1983, George Gargov christened the name to Intuitionistic Fuzzy Sets (IFS), which does not include the law of excluded middle. Thus the concept of Intuitionistic Fuzzy Sets was developed which gave the scope for inclusion of degree of hesitancy $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$. On the basis of Intuitionistic fuzzy set, Atanassov and Gargov introduced the concept of Interval-valued Intuitionistic Fuzzy set, where the membership and non-membership functions are all intervals which has more practical applications [1] [2]. Furthermore, the concepts of cut sets and fuzzy numbers were developed and intensive research has been taking place around the world in different areas. Researchers have proposed;

1. Arithmetic operations on Fuzzy numbers with and without α-cuts, see [5].

2. Arithmetic operations on Fuzzy numbers with interval cuts and their corresponding representation and decomposition theorems, see [17].

3. Intuitionistic Fuzzy numbers with (α, β)-cuts.

4. Representation theorems and Decomposition theorems for Interval cut sets on Interval-valued Intuitionistic Fuzzy Sets, see [17].

The symmetric form of α-cut, inverse α-cut and then α-induced fuzzy set have been examined along with their properties[14]. However, the progress in the cut-set area have been stopped at the point of introducing interval cut-sets in fuzzy sets and numbers. The single cuts can be gradually improvised to many cuts being introduced at the same time which in turn reduces the number of elements in the universe and helps in working in a target area. In this paper we have attempted to introduce four cuts in the form of matrix-cut. The resultant cut-matrix will have elements in the matrix format. This concept has been developed keeping in mind the essence of introducing α-cuts in fuzzy sets and fuzzy numbers. We are also extending the concept of matrix cut to Intuitionistic Fuzzy sets and Interval-valued Intuitionistic Fuzzy sets. An attempt has been made to display multi-data set at the same time.

2. Preliminaries

Definition. *Fuzzy subset*: Let E be a set, denumerable or not, and let x be an element of E. Then a fuzzy subset $\widetilde{\mathrm{A}}$ of E is a set of ordered pairs $\{(x, \mu_A(x)), \forall x \in E\}$, where $\mu_A(x)$ is the grade or degree of membership of x in $\widetilde{\mathrm{A}}$[9].

Definition. *Ordinary subset of level α:* Let $\alpha \in [0,1]$; then the ordinary set $A_\alpha = \left\{x/\mu_{\widetilde{\mathrm{A}}}(x) \geq \alpha\right\}$is called the ordinary subset of level α [9].

Definition *Interval-valued level cut sets on fuzzy sets:* Let E be a set and $3^E = \{A/A : E \to \{0, \frac{1}{2}, 1\}$ is a mapping. Then 3^E is a F-Lattice. Let F(E) be a set of all fuzzy subsets of E. Then if $A \in F(E)$ and $\alpha = [a_1, a_2] \in \bar{L}$, the three-valued cut set has been defined as,

$$A_\alpha = \begin{cases} 1, \; A(x) \geq a_2 \\ \frac{1}{2}, \, a_1 \leq A(x) \leq a_2 \\ 0, \; A\left(x\right) < a_1 \end{cases}$$

Definition. *Intuitionistic Fuzzy Set (IFS):* An IFS A in E is defined as an object of the following form, $A = \{(x, \mu_A(x), \upsilon_A(x))/x \in E\}$,where $\mu_A : E \to [0,1]$ and $\upsilon_A : E \to [0,1]$define the degree of membership and degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$: $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$. The value of $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ is called the degree of hesitancy or uncertainty of the element $x \in E$ to the Intuitionistic Fuzzy set A.

Definition. A set of (α, β)-level, generated by an IFS A, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$, is defined as

$$N_{\alpha,\beta}(A) = \{(x, \mu_A(x), \upsilon_A(x))/x \in E, \mu_A(x) \geq \alpha, \upsilon_A(x) \leq \beta\},$$

see [1].

Definition. *Interval cut sets*: Let $A = \{(x, \mu_A(x), \upsilon_A(x))/x \in E\}$be an Intuitionistic fuzzy set and $a \in [0,1]$.Then we call

$$A_a(x) = \begin{cases} 1, \; \mu_A(x) \geq a \\ \frac{1}{2}, \, \mu_A(x) < a \leq 1 - \upsilon_A(x) \\ 0, \; a > 1 - \upsilon_A(x) \end{cases},$$

the a-upper cut set of A [17]. **Definition** Let $A = \{(x, \mu_A(x), \upsilon_A(x))/x \in E\}$be an Intuitionistic fuzzy set and $a \in [0,1]$. Then we call

$$A_{\underline{a}}(x) = \begin{cases} 1, \; \mu_A(x) > a \\ \frac{1}{2}, \, \mu_A(x) \leq a < 1 - \upsilon_A(x) \\ 0, \; a \geq 1 - \upsilon_A(x) \end{cases}$$ the strong a-upper cut set of A.

Definition Let $A = \{(x, \mu_A(x), \upsilon_A(x)) / x \in E\}$be an Intuitionistic fuzzy set and $a \in [0,1]$. Then we define the following:

1. $A^a(x) = \begin{cases} 1, \ \upsilon_A(x) \geq a \\ \frac{1}{2}, \upsilon_A(x) < a < 1 - \mu_A(x) \\ 0, \ a \geq 1 - \mu_A(x) \end{cases}$
   the a-lower cut set of A.

2. $A^{\underline{a}}(x) = \begin{cases} 1, \ \upsilon_A(x) > a \\ \frac{1}{2}, \upsilon_A(x) \leq a < 1 - \mu_A(x) \\ 0, \ a \geq 1 - \mu_A(x) \end{cases}$
   the Strong a-lower cut set of A.

3. $A_{[a]}(x) = \begin{cases} 1, \ \mu_A(x) + a \geq 1 \\ \frac{1}{2}, \upsilon_A(x) \leq a < 1 - \mu_A(x) \\ 0, \ a < \upsilon_A(x) \end{cases}$
   the a-upper quasi cut set of A.

4. $A_{[\underline{a}]}(x) = \begin{cases} 1, \ \mu_A(x) + a > 1 \\ \frac{1}{2}, \upsilon_A(x) < a \leq 1 - \mu_A(x) \\ 0, \ a \leq \upsilon_A(x) \end{cases}$
   the Strong a-upper quasi cut set of A.

5. $A_{[\underline{a}]}(x) = \begin{cases} 1, \ \mu_A(x) + a > 1 \\ \frac{1}{2}, \upsilon_A(x) < a \leq 1 - \mu_A(x) \\ 0, \ a \leq \upsilon_A(x) \end{cases}$
   the Strong a-upper quasi cut set of A.

6. $A^{[a]}(x) = \begin{cases} 1, \ a + \upsilon_A^-(x) \geq 1 \\ \frac{3}{4}, \upsilon_A^-(x) < 1 - a \leq \upsilon_A^+(x) \\ \frac{1}{2}, \ \upsilon_A^+(x) \leq 1 - a \leq 1 - \ \mu_A^+(x) \\ \frac{1}{4}, \mu_A^-(x) \leq a < \mu_A^+(x) \\ 0, \mu_A^-(x) > a \end{cases}$
   the a-lower quasi cut set of A.

7. $A^{[\underline{a}]}(x) = \begin{cases} 1, \ a + \upsilon_A^-(x) > 1 \\ \frac{3}{4}, \upsilon_A^-(x) \leq 1 - a < \upsilon_A^+(x) \\ \frac{1}{2}, \ \upsilon_A^+(x) \leq 1 - a < 1 - \ \mu_A^+(x) \\ \frac{1}{4}, \mu_A^-(x) < a \leq \mu_A^+(x) \\ 0, \mu_A^-(x) \geq a \end{cases}$
   the Strong a-lower quasi cut set of A.

3. Proposed Definitions and Theorems

Definition. (Matrix Cut) Let $\Delta = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ with $0 \leq \alpha \leq 1$,$0 \leq \beta \leq 1$, $0 \leq \gamma \leq 1$, $0 \leq \delta \leq 1$ and $\Delta^c = \begin{bmatrix} 1-\alpha & 1-\beta \\ 1-\gamma & 1-\delta \end{bmatrix}$ where $1-\alpha \in [0,1]$, $1-\beta \in [0,1]$, $1-\gamma \in [0,1]$, $1-\delta \in [0,1]$.
Then we define Δ-lower Matrix cut set as $\tilde{A}_\Delta(x) = \begin{bmatrix} \tilde{A}_\alpha(x) & \tilde{A}_\beta(x) \\ \tilde{A}_\gamma(x) & \tilde{A}_\delta(x) \end{bmatrix}$ where $\tilde{A}_\alpha$ is the α-cut set of $\tilde{A}$. Similarly,$\tilde{A}_\beta$,$\tilde{A}_\gamma$ and $\tilde{A}_\delta$ are the β, γ and δ cut sets respectively of $\tilde{A}$.
The given format can be used to represent the information in an array or matrix form of any size. The development of fuzzy matrix cut $\tilde{A}_\Delta(x)$ are as follows;

If $\tilde{A}_\Delta(x)$ is a fuzzy set with the Δ-upper matrix cut then we define,

1. $\tilde{A}_{\underline{\Delta}}(x) = \begin{bmatrix} \tilde{A}_{\underline{\alpha}}(x) & \tilde{A}_{\underline{\beta}}(x) \\ \tilde{A}_{\underline{\gamma}}(x) & \tilde{A}_{\underline{\delta}}(x) \end{bmatrix}$ as the strong Δ-upper matrix cut where

$$\tilde{A}_{\underline{\alpha}}(x) = \left\{ x/\mu_{\underset{\sim}{\mathrm{A}}}(x) > \alpha, \beta, \gamma, \delta \right\}$$

2. $\tilde{A}^\Delta(x) = \begin{bmatrix} \tilde{A}^\alpha(x) & \tilde{A}^\beta(x) \\ \tilde{A}^\gamma(x) & \tilde{A}^\delta(x) \end{bmatrix}$as the Δ-lower matrix cut where

$$\tilde{A}^\beta(x) = \left\{ x/\mu_{\underset{\sim}{\mathrm{A}}}(x) \leq \beta \right\}.$$

3. $\tilde{A}^{\underline{\Delta}}(x) = \begin{bmatrix} \tilde{A}^{\underline{\alpha}}(x) & \tilde{A}^{\underline{\beta}}(x) \\ \tilde{A}^{\underline{\gamma}}(x) & \tilde{A}^{\underline{\delta}}(x) \end{bmatrix}$as the strong$\Delta$-lower matrix cut

$$\tilde{A}^{\underline{\beta}}(x) = \left\{ x/\mu_{\underset{\sim}{\mathrm{A}}}(x) < \beta \right\}$$

4. $\tilde{A}_{[\Delta]}(x) = \begin{bmatrix} \tilde{A}_{[\alpha]}(x) & \tilde{A}_{[\beta]}(x) \\ \tilde{A}_{[\gamma]}(x) & \tilde{A}_{[\delta]}(x) \end{bmatrix}$ as the Δ-quasi upper matrix cut where

$$\tilde{A}_{[\gamma]}(x) = \left\{ x \in E/\mu_{\underset{\sim}{A}}(x) + \gamma \geq 1 \right\}$$

5. $\tilde{A}_{[\underline{\Delta}]}(x) = \begin{bmatrix} \tilde{A}_{[\underline{\alpha}]}(x) & \tilde{A}_{[\underline{\beta}]}(x) \\ \tilde{A}_{[\underline{\gamma}]}(x) & \tilde{A}_{[\underline{\delta}]}(x) \end{bmatrix}$ as the strong Δ-quasi upper matrix cut where

$$\tilde{A}_{[\underline{\gamma}]}(x) = \left\{ x \in E/\mu_{\underset{\sim}{A}}(x) + \gamma > 1 \right\}.$$

6. $\tilde{A}^{[\Delta]}(x) = \begin{bmatrix} \tilde{A}^{[\alpha]}(x) & \tilde{A}^{[\beta]}(x) \\ \tilde{A}^{[\gamma]}(x) & \tilde{A}^{[\delta]}(x) \end{bmatrix}$ as the Δ-quasi lower matrix cut where

$$\tilde{A}^{[\delta]}(x) = \left\{ x/\mu_{\underset{\sim}{A}}(x) + \delta \leq 1 \right\}$$

7. $\tilde{A}^{[\underline{\Delta}]}(x) = \begin{bmatrix} \tilde{A}^{[\underline{\alpha}]}(x) & \tilde{A}^{[\underline{\beta}]}(x) \\ \tilde{A}^{[\underline{\gamma}]}(x) & \tilde{A}^{[\underline{\delta}]}(x) \end{bmatrix}$ as the strong Δ-quasi lower matrix cut where $\tilde{A}^{[\underline{\delta}]}(x) = \left\{ x/\mu_{\underset{\sim}{A}}(x) + \delta < 1 \right\}$.

Note: The property of the matrix cut depends on the property of each and every element defined in the matrix.

3.1. Properties of Matrix Cut

1. $\tilde{A}_{\underline{\Delta}} \subset \tilde{A}_{\Delta}$
2. $\tilde{A} \subset \tilde{B} \Rightarrow \tilde{A}_{\Delta} \subset \tilde{B}_{\Delta}, \tilde{A}_{\underline{\Delta}} \subset \tilde{B}_{\underline{\Delta}}$.
3. $\left(\tilde{A}^c\right)_{\Delta} = \left(\tilde{A}_{\underline{\Delta}^c}\right)^c, \left(\tilde{A}^c\right)_{\underline{\Delta}} = \left(\tilde{A}_{\Delta^c}\right)^c$.
4. $\left(\tilde{A} \bigcup \tilde{B}\right)_{\Delta} = \tilde{A}_{\Delta} \bigcup \tilde{B}_{\Delta}, \left(\tilde{A} \bigcap \tilde{B}\right)_{\Delta} = \tilde{A}_{\Delta} \bigcap \tilde{B}_{\Delta}$
5. $\bigcup_{i\in I} \left(\tilde{A}_i\right)_{\Delta} \subset \left(\bigcup_{i\in I} \tilde{A}_i\right)_{\Delta}, \bigcap_{i\in I} \left(\tilde{A}_i\right)_{\Delta} = \left(\bigcap_{i\in I} \tilde{A}_i\right)_{\Delta}$

3.2. Arithmetic Operations on Fuzzy Numbers using the Δ-Matrix Cut

Let $\tilde{A} = (a, b, c, d)$ and $\tilde{P} = (p, q, r, s)$ be two trapezoidal fuzzy numbers. Then the arithmetic operations on the matrix cut will be established using interval arithmetic. For the sake of convenience, we consider a single interval (α, β) and study the arithmetic operations on them and apply the same to every interval obtained by the matrix cut.

3.2.1. Addition of two Trapezoidal Fuzzy Numbers

Let $\tilde{A} = (a,b,c,d)$ and $\tilde{P} = (p,q,r,s)$ be two Trapezoidal Fuzzy Numbers. Then the sum is given by,

$$\mu_{\tilde{A}+\tilde{P}}(x) = \begin{cases} \left(\frac{x_1-(a+p)}{(b+q)-(a+p)}, \frac{x_2-(a+p)}{(b+q)-(a+p)}\right), & a+p \le x_1 < x_2 \le b+q \\ 1, & b+q \le x_1 < x_2 \le c+r \\ \left(\frac{(d+s)-x_1}{(d+s)-(c+r)}, \frac{(d+s)-x_2}{(d+s)-(c+r)}\right), & c+r \le x_1 < x_2 \le d+s \\ 0, & otherwise \end{cases}$$

3.2.2. Difference of two Trapezoidal Fuzzy Numbers

Let $\tilde{A} = (a,b,c,d)$ and $\tilde{P} = (p,q,r,s)$ be two trapezoidal fuzzy numbers. Then the difference is given by,

$$\mu_{\tilde{A}-\tilde{P}}(x) = \begin{cases} \left(\frac{x_1-(a-s)}{(b-r)-(a-s)}, \frac{x_2-(a-s)}{(b-r)-(a-s)}\right), & a-s \le x_1 < x_2 \le b-r \\ 1, & b-r \le x_1 < x_2 \le c-q \\ \left(\frac{(d-p)-x_1}{(d-p)-(c-q)}, \frac{(d-p)-x_2}{(d-p)-(c-q)}\right), & c-q \le x_1 < x_2 \le d-p \\ 0, & otherwise \end{cases}$$

3.2.3. Product of two Trapezoidal Fuzzy Numbers

Let $\tilde{A} = (a,b,c,d)$ and $\tilde{P} = (p,q,r,s)$ be two trapezoidal fuzzy numbers. Then the product is given by,

$$\mu_{\tilde{A}-\tilde{P}}(x) = \begin{cases} \left(\frac{\sqrt{4Bx_1-4BR+Q^2}-Q}{2B}, \frac{\sqrt{4Bx_2-4BR+Q^2}-Q}{2B}\right), & ap \le x_1 < x_2 \le bq \\ 1, & bq \le x_1 < x_2 \le cr \\ \left(\frac{\sqrt{4B'x_1-4B'R'+Q'^2}-Q'}{2P'}, \frac{\sqrt{4B'x_2-4B'R'+Q'^2}-Q'}{2P'}\right), & cr \le x_1 < x_2 \le ds \\ 0, & otherwise \end{cases}$$

Where

$$B = (b-a)(q-p), \; Q = p(b-a) + a(q-p), \; R = ap$$

$$B' = (d-c)(s-r), \; Q' = s(c-d) + d(r-s), \; R' = ds$$

3.2.4. Quotient of two Trapezoidal Fuzzy Numbers

Let $\tilde{A} = (a, b, c, d)$ and $\tilde{P} = (p, q, r, s)$ be two trapezoidal fuzzy numbers. Then the sum is given by,

$$\mu_{\tilde{A}*\tilde{P}}(x) = \begin{cases} \left(\frac{a-px_1}{x_1(p-q)-(b-a)}, \frac{a-px_2}{x_1(p-q)-(b-a)}\right), & \frac{a}{s} \leq x_1 < x_2 \leq \frac{b}{r} \\ 1, & \frac{b}{r} \leq x_1 < x_2 \leq \frac{c}{q} \\ \left(\frac{d-x_1 s}{(d-c)-x_1(s-r)}, \frac{d-x_2 s}{(d-c)-x_2(s-r)}\right), & \frac{c}{q} \leq x_1 < x_2 \leq \frac{d}{p} \\ 0, & otherwise \end{cases}$$

We have thus given the basic arithmetic operations on trapezoidal numbers using a single interval cut. This concept will then be applied to every interval under consideration to obtain the required results.

3.3. Decomposition Theorems

Theorem 1. First Decomposition Theorem: *Let E be a universe and $\tilde{A}$ be a fuzzy subset. Then the first decomposition theorem for the Δ- upper matrix cut and strong Δ-upper matrix cut on fuzzy subsets can be written as,*

1. $A = \bigcup_\Delta \Delta\tilde{A}_\Delta$, $A^c = \bigcap_\Delta \Delta^c\tilde{A}_\Delta$
2. $A = \bigcup_\Delta \Delta\tilde{A}_{\underline{\Delta}}$, $A^c = \bigcap_\Delta \Delta^c\tilde{A}_{\underline{\Delta}}$

Theorem 2. Second Decomposition Theorem:*Let E be a universe and $\tilde{A}$ be a fuzzy subset. Then the second decomposition theorem for the Δ- quasi upper matrix cut and strong Δ- quasi upper matrix cut on fuzzy subsets can be written as,*

1. $A = \bigcup_\Delta \Delta^c\tilde{A}_{[\Delta]}$, $A^c = \bigcap_\Delta \Delta\tilde{A}_{[\Delta]}$
2. $A = \bigcup_\Delta \Delta^c\tilde{A}_{[\underline{\Delta}]}$, $A^c = \bigcap_\Delta \Delta\tilde{A}_{[\underline{\Delta}]}$

Theorem 3. Third Decomposition Theorem: *Let E be a universe and $\tilde{A}$ be a fuzzy subset. Then the third decomposition theorem for the Δ- lower matrix cut and strong Δ- lower matrix cut on fuzzy subsets can be written as,*

1. $A^c = \bigcup_\Delta \Delta^c\tilde{A}^\Delta$, $A = \bigcap_\Delta \Delta\tilde{A}^\Delta$
2. $A^c = \bigcup_\Delta \Delta^c\tilde{A}^{\underline{\Delta}}$, $A = \bigcap_\Delta \Delta\tilde{A}^{\underline{\Delta}}$

Theorem 4. Fourth Decomposition Theorem:*Let E be a universe and $\tilde{A}$ be a fuzzy subset. Then the fourth decomposition theorem for the Δ-quasi lower matrix cut and strong Δ- quasi lower matrix cut on fuzzy subsets can be written as,*

1. $A^c = \bigcup_\Delta \Delta \tilde{A}^{[\Delta]}$, $A = \bigcap_\Delta \Delta^c \tilde{A}^{[\Delta]}$
2. $A^c = \bigcup_\Delta \Delta \tilde{A}^{[\underline{\Delta}]}$, $A = \bigcap_\Delta \Delta^c \tilde{A}^{[\underline{\Delta}]}$

3.4. Representation of Interval Type Cut Sets

Definition. The proposed matrix cut can also be visualized in the following form in terms of interval level cut set;

$$\tilde{A}_\Delta = \left\{ \begin{array}{c} \begin{bmatrix} \tilde{A}_{(\alpha,\beta)} & \tilde{A}_{(\alpha,\gamma)} \\ \tilde{A}_{(\alpha,\delta)} & \tilde{A}_{(\beta,\gamma)} \\ \tilde{A}_{(\beta,\delta)} & \tilde{A}_{(\gamma,\delta)} \end{bmatrix} / \begin{bmatrix} \mu_{\tilde{A}}(x) \in (\alpha,\beta) & \mu_{\tilde{A}}(y) \in (\alpha,\gamma) \\ \mu_{\tilde{A}}(z) \in (\alpha,\delta) & \mu_{\tilde{A}}(u) \in (\beta,\gamma) \\ \mu_{\tilde{A}}(v) \in (\beta,\delta) & \mu_{\tilde{A}}(w) \in (\gamma,\delta) \end{bmatrix} \\ with \left. \begin{array}{c} (\alpha,\beta) \\ (\alpha,\gamma) \\ (\alpha,\delta) \\ (\beta,\gamma) \\ (\beta,\delta) \\ (\gamma,\delta) \end{array} \right\} \subset [0,1] \end{array} \right\}$$

Definition.

1. $\_\tilde{A}_{\underline{\Delta}} = \begin{bmatrix} \tilde{A}_{(\underline{\alpha},\underline{\beta})} & \tilde{A}_{(\underline{\alpha},\underline{\gamma})} \\ \tilde{A}_{(\underline{\alpha},\underline{\delta})} & \tilde{A}_{(\underline{\beta},\underline{\gamma})} \\ \tilde{A}_{(\underline{\beta},\underline{\delta})} & \tilde{A}_{(\underline{\gamma},\underline{\delta})} \end{bmatrix}$

where each cut can be categorized in three ways;

1. $\tilde{A}_{(\underline{\alpha}_1,\underline{\beta}_1)} = \{x \in E/\alpha < \mu_{\tilde{A}}(x) < \beta\}$
2. $\tilde{A}_{(\underline{\alpha}_2,\underline{\beta}_2)} = \{x \in E/\alpha \leq \mu_{\tilde{A}}(x) < \beta\}$
3. $\tilde{A}_{(\underline{\alpha}_3,\underline{\beta}_3)} = \{x \in E/\alpha < \mu_{\tilde{A}}(x) \leq \beta\}$

The lower cuts, strong lower cuts are defined in the similar fashion. But, a thorough understanding of the quasi cut has to be discussed in case of matrix cuts.

Definition. If $\tilde{A}_{\Delta}(x)$ is a fuzzy set with the Δ-upper matrix cut then we define,

$\tilde{A}_{[\Delta]}(x) = \left\{x \in E/\mu_{\underset{A}{\sim}}(x) + \alpha + \beta \geq \frac{1}{2}\right\}$as the Δ-quasi upper matrix cut.

$\tilde{A}_{[\underset{\sim}{\gamma}]}(x) = \left\{x \in E/\mu_{\underset{A}{\sim}}(x) + \alpha + \beta > \frac{1}{2}\right\}$as the strong Δ-quasi upper matrix cut

$\tilde{A}^{[\Delta]}(x) = \left\{x/\mu_{\underset{A}{\sim}}(x) + \alpha + \beta \leq \frac{1}{2}\right\}$as the Δ-quasi lower matrix cut and,

$\tilde{A}^{[\Delta]}(x) = \left\{x/\mu_{\underset{A}{\sim}}(x) + \alpha + \beta < \frac{1}{2}\right\}$as the Strong Δ-quasi lower matrix cut.

Furthermore, every interval defined under the interval matrix cut will have the above mentioned definitions.

3.5. Representation of Interval cut sets in Intuitionistic Fuzzy Number

Definition. Let $\Delta = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ be the proposed matrix cut with $A_{\Delta} = \begin{bmatrix} A_{(\alpha_1,\alpha_2)} & A_{(\beta_1,\beta_2)} \\ A_{(\gamma_1,\gamma_2)} & A_{(\delta_1,\delta_2)} \end{bmatrix}$, where

$$A_{(\alpha_1,\alpha_2)} = \{x/\mu_A(x) \geq \alpha_1, \upsilon_A(x) \leq \alpha_2\}$$

$$A_{(\beta_1,\beta_2)} = \{x/\mu_A(x) \geq \beta_1, \upsilon_A(x) \leq \beta_2\}$$

$$A_{(\gamma_1,\gamma_2)} = \{x/\mu_A(x) \geq \gamma_1, \upsilon_A(x) \leq \gamma_2\}$$

$$A_{(\delta_1,\delta_2)} = \{x/\mu_A(x) \geq \delta_1, \upsilon_A(x) \leq \delta_2\}$$

The proposed matrix cut can be represented as follows,
$A_{\Delta} = \{x \in E/\alpha \leq \mu_A(x) \leq \gamma, \delta \leq \upsilon_A(x) \leq \beta\}$with $0 \leq \alpha + \delta < \gamma + \beta \leq 1$. The role of cuts α, β, γ and δ can be interchanged according to the convenience and the results required. The membership function will be redefined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x_\alpha - a}{b-a}, \frac{x_\gamma - a}{b-a}\right) & , (x_\alpha, x_\gamma) \subset (a,b) \\ 1 & , (x_\alpha, x_\gamma) \subset (b,c) \\ \left(\frac{d-x_\alpha}{d-c}, \frac{d-x_\gamma}{d-c}\right) & , (x_\alpha, x_\gamma) \subset (c,d) \\ 0 & , otherwise \end{cases}$$

$$\upsilon_{\tilde{A}}(x) = \begin{cases} \left(\frac{b-x_\delta}{b-a'}, \frac{b-x_\beta}{b-a'}\right), & (x_\delta, x_\beta) \subset (a', b) \\ 0 & ,(x_\delta, x_\beta) \subset (b, c) \\ \left(\frac{x_\delta - c}{d'-c}, \frac{x_\beta - c}{d'-c}\right), & (x_\delta, x_\beta) \subset (c, d') \\ 1 & , otherwise \end{cases}$$

3.6. Arithmetic Operations on Intuitionistic Fuzzy Numbers Based on Matrix Cut

Let $\tilde{A}^i = (a, b, c, d; a', b, c, d')$ and $\tilde{P}^i = (p, q, r, s; p', q, r, s')$ be two intuitionistic trapezoidal fuzzy numbers. Then we give the arithmetic operations on the two Intuitionistic Fuzzy Numbers based on our proposed matrix cut method.

3.6.1. Sum of Two Trapezoidal Intuitionistic Fuzzy Numbers

Let $\tilde{A}^i = (a, b, c, d; a', b, c, d')$ and $\tilde{P}^i = (p, q, r, s; p', q, r, s')$ be two Trapezoidal Intuitionistic Fuzzy Numbers. Then the sum of the two numbers is given by, $\tilde{S}^i = \tilde{A}^i \oplus \tilde{P}^i$. The membership and non-membership functions are given as follows;

$$\mu_{\tilde{S}^i}(x) = \begin{cases} \left(\frac{x_1-(a+p)}{(b-a)+(q-p)}, \frac{x_2-(a+p)}{(b-a)+(q-p)}\right), & a+p \leq x_1 < x_2 \leq b+q \\ 1, & b+q \leq x_1 < x_2 \leq c+r \\ \left(\frac{d+s-x_1}{(d-c)+(s-r)}, \frac{d+s-x_2}{(d-c)+(s-r)}\right), & c+r \leq x_1 < x_2 \leq d+s \\ 0, & otherwise \end{cases}$$

$$\upsilon_{\tilde{S}^i}(x) = \begin{cases} \left(\frac{b+q-x_1}{(b-a')+(q-p')}, \frac{b=q-x_2}{(b-a')+(q-p')}\right), & a'+p' \leq x_1 < x_2 \leq b+q \\ 0, & b+q \leq x_1 < x_2 \leq c+r \\ \left(\frac{x_1-(c+r)}{(d'-c)+(s'-r)}, \frac{x_2-(c+r)}{(d'-c)+(s'-r)}\right), & c+r \leq x_1 < x_2 \leq d'+s' \\ 1, & otherwise \end{cases}$$

3.6.2. Product of two Trapezoidal Intuitionistic Fuzzy Numbers

Let $\tilde{A}^i = (a, b, c, d; a', b, c, d')$ and $\tilde{P}^i = (p, q, r, s; p', q, r, s')$ be two Trapezoidal Intuitionistic Fuzzy Numbers. Then the product is given by

$$\mu_{\tilde{Q}^i}(x) = \begin{cases} \left(\frac{-R_1+\sqrt{R_1^2-4T_1(ap-x_1)}}{2T_1}, \frac{-R_1+\sqrt{R_1^2-4T_1(ap-x_2)}}{2T_1}\right), & ap \le x_1 < x_2 \le bq \\ 1, & bq \le x_1 < x_2 \le cr \\ \left(\frac{R_2-\sqrt{R_2^2-4T_2(ds-x_1)}}{2T_2}, \frac{R_2-\sqrt{R_2^2-4T_2(ds-x_2)}}{2T_2}\right), & cr \le x_1 < x_2 \le ds \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{Q}^i}(x) = \begin{cases} \left(1 - \frac{-R'_1+\sqrt{R'^2_1-4T'_1(a'p'-x_1)}}{2T'_1}, 1 - \frac{-R'_1+\sqrt{R'^2_1-4T'_1(a'p'-x_2)}}{2T'_1}\right), \\ \quad a'p' \le x_1 < x_2 \le bq \\ 0, \ bq \le x_1 < x_2 \le cr \\ \left(1 - \frac{R'_2-\sqrt{R'^2_2-4T'_2(d's'-x_1)}}{2T'_2}, 1 - \frac{R'_2-\sqrt{R'^2_2-4T'_2(d's'-x_2)}}{2T'_2}\right), \\ \quad cr \le x_1 < x_2 \le d's' \\ 1, \ otherwise \end{cases}$$

where

$$T_1 = (b-a)(q-p),$$
$$T_2 = (d-c)(s-r),$$
$$R_1 = p(b-a) + a(q-p), \quad R_2 = -(s(d-c) + d(s-r)),$$
$$T'_1 = (b-a')(q-p'), \quad T'_2 = (d'-c)(s'-r),$$
$$R'_1 = p'(b-a') + a'(q-p'),$$
$$R'_2 = -(s'(d'-c) + d'(s'-r)).$$

3.6.3. Quotient of two Trapezoidal Intuitionistic Fuzzy Numbers

Let $\tilde{A}^i = (a, b, c, d; a', b, c, d')$ and $\tilde{P}^i = (p, q, r, s; p', q, r, s')$ be two intuitionistic trapezoidal fuzzy numbers. Then the quotient of the two numbers can be approximated to, $\tilde{D}^i = \tilde{A}^i \div \tilde{P}^i$. The membership and non-membership functions are given as follows;

$$\mu_{\tilde{D}^i}(x) = \begin{cases} \left(\frac{sx_1-a}{(b-a)+x_1(s-r)}, \frac{sx_2-a}{(b-a)+x_2(s-r)}\right), & \frac{a}{p} \le x_1 < x_2 \le \frac{b}{r} \\ 1, & \frac{b}{r} \le x_1 < x_2 \le \frac{c}{q} \\ \left(\frac{a-px_1}{(d-c)+x_1(q-p)}, \frac{a-px_2}{(d-c)+x_2(q-p)}\right), & \frac{c}{q} \le x_1 < x_2 \le \frac{d}{p} \\ 0, & otherwise \end{cases}$$

$$\upsilon_{\tilde{D}^i}(x) = \begin{cases} \left(\frac{b-rx_1}{(b-a')+x_1(s'-r)}, \frac{b-rx_2}{(b-a')+x_2(s'-r)}\right), & \frac{a'}{p'} \leq x_1 < x_2 \leq \frac{b}{r} \\ 0, & \frac{b}{r} \leq x_1 < x_2 \leq \frac{c}{q} \\ \left(\frac{q-rx_1}{(d'-c)+x_1(q-p)}, \frac{q-rx_2}{(d'-c)+x_2(q-p)}\right), & \frac{c}{q} \leq x_1 < x_2 \leq \frac{d'}{p'} \\ 1, & otherwise \end{cases}$$

4. Conclusion

In this paper, we have contributed a new perspective of cut sets called the matrix cut. Based on this, Decomposition theorems and Arithmetic operations on Fuzzy sets, Trapezoidal Fuzzy Numbers and Trapezoidal Intuitionistic Fuzzy Numbers have been discussed. The proposed cut will be further studied to prove ranking and other broad application areas in Fuzzy sets and Intuitionistic fuzzy sets. The proposed method can be used extensively in Pentagonal Fuzzy numbers and Pentagonal Intuitionistic Fuzzy Numbers, Hexagonal Fuzzy numbers and so on.

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