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THE GENERALIZED HYERS-ULAM FUZZY STABILITY OF CUBIC FUNCTIONAL EQUATION

R. Murali[1 §], V. Vithya[2]
[1,2]Department of Mathematics
Sacred Heart College
Tirupattur, 635 601, Tamilnadu, INDIA

Abstract: In this paper, we prove the generalized Hyers-Ulam stability for the generalized cubic functional equation in matrix fuzzy normed space using fixed point method.

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1. Introduction and Preliminaries

The first stability problem concerning group homomorphisms was raised by Ulam [15] in 1940 and affirmatively solved by Hyers [7]. The result of Hyers was generalized by Aoki [1] for approximate additive mappings and by Rassias [14] for approximate linear mappings by allowing the difference Cauchy equation $\|f(x+y)-f(x)-f(y)\|$ to be controlled by $\epsilon(\|x\|^p+\|y\|^p)$. Taking into consideration a lot of influence of Ulam, Hyers and Rassias on the development of stability problems of functional equations, the stability phenomenon that was proved by Rassias is called the Hyers-Ulam-Rassias stability. In 1994, a generalization of Rassias's theorem was obtained by Gavruta [6], who replaced

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§Correspondence author

$\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\phi(x, y)$. Since then the stability problems of various functional equations and mappings, the cubic equation, homomorphisms, derivations and their pexiderized versions with more general domains and ranges have been investigated by a number of authors.

Definition 1. (see [13]) Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a generalized metric on X if d satisfies

1. $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$ for all $x, y \in X$;
3. $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

we recall a fundamental result in fixed point theory.

Theorem 2. (see [13]) *Let (X, d) be a complete generalized metric space and $J : X \to Y$ be a strictly contractive mapping with a Lipschitz constant $\alpha < 1$. Then, for each given element $x \in X$, either $d(J^n x, J^{n+1}x) = \infty$, for all nonnegative integer n or there exists a positive integer n_0 such that*

1. *$d(J^n x, J^{n+1}x) < \infty$, for all $n \geq n_0$;*
2. *the sequence $\{J^n x\}$ converges to a fixed point $y*$ of J;*
3. *$y*$ is the unique fixed point of J in the set $Y = \{y \in X | d(J^{n_0} x, y) < \infty\}$;*
4. *$d(y, y*) \leq \frac{1}{1-\alpha} d(y, Jy)$ for all $y \in Y$.*

In 1996, Isac and Rassias were the first to provide applications of stability theory of functional equations for the proof of new fixed point theorems with applications. By using fixed point methods, the stability problems of several functional equations have been extensively investigated by a number of authors.

Katsaras [8] defined a fuzzy norm on a vector space to construct a fuzzy vector topological structure on the space. Some mathematicians have defined fuzzy norms on a vector space from various points of view [5, 10]. In particular, Bag and Samanta [2], following Cheng and Mordeson [4], gave an idea of a fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [9]. They established a decomposition theorem of a fuzzy norm into a family of crisp norms and investigated some properties of fuzzy normed spaces [3].

We use the definition of fuzzy normed spaces given in [2, 13] to investigate the generalized Hyers-Ulam stability for the cubic functional equation in the fuzzy normed vector space setting.

Definition 3. (see [2, 13]) Let X be a real vector space. A function $N : X \times \mathbb{R} \to [0, 1]$ is called a fuzzy norm on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$:

1. (i) $N(x, t) = 0$ for $t \leq 0$;
2. (ii) $x = 0 \Leftrightarrow N(x, t) = 1$ for all $t > 0$;
3. (iii) $N(cx, t) = N(x, \frac{t}{|c|})$ if $c \neq 0$;
4. (iv) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$;
5. (v) $N(x, .)$ is a non-decreasing function of $\mathbb{R}$ and $\lim_{t\to\infty} N(x, t) = 1$;
6. (vi) For $x \neq 0$, $N(x, .)$ is continuous on $\mathbb{R}$.

The pair (X, N) is called a fuzzy normed vector space.

Definition 4. (see [2, 13]) Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to be convergent or converge if there exist an $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case, x is called the limit of the sequence $\{x_n\}$ and we denote it by $N - \lim_{n\to\infty} x_n = x$.

Definition 5. (see [2, 13]) Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to be Cauchy if for each $\epsilon > 0$ and each $t > 0$ there exist an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, t) > 1 - \epsilon$.

It is well known that every convergent sequence in a fuzzy normed vector space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed vector space is called a fuzzy Banach space.

We say that a mapping $f : X \to Y$ between fuzzy normed vector space X and Y is continuous at a point $x_0 \in X$ if for each sequence $\{x_n\}$ converging to $x_0 \in X$, the sequence $\{f(x_n)\}$ converges to $f(x_0)$. If $f : X \to Y$is continuous at each $x \in X$, then $f : X \to Y$ is said to be continuous on X.

We will use the following notations:

$M_n(X)$ is the set of all $n \times n$-matrices in X;

$e_j \in M_{1,n}(\mathbb{C})$ is that jth component is 1 and the other components are zero;

$E_{ij} \in M_n(\mathbb{C})$ is that (i,j)-component is 1 and the other components are zero;

$E_{ij} \otimes x \in M_n(X)$ is that (i,j)-component is x and the other components are zero.

For $x \in M_n(X), y \in M_k(X)$,

$$x \oplus y = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}.$$

Note that $(X, \{\|.\|_n\})$ is a matrix normed space if and only if $(M_n(X), \|.\|_n)$ is a normed space for each positive integer n and $\|AxB\|_k \leq \|A\| \, \|B\| \, \|x\|_n$ holds for $A \in M_{k,n}(\mathbb{C})$, $B \in M_{n,k}(\mathbb{C})$ and $x = (x_{ij}) \in M_n(X)$, and that $(X, \{\|.\|_n\})$ is a matrix Banach space if and only if X is a Banach space and $(X, \{\|.\|_n\})$ is a matrix normed space. A matrix normed space $(X, \{\|.\|_n\})$ is called an L^∞ -matrix normed space if $\|x \otimes y\|_{n+k} = \max\{\|x\|_n, \|y\|_k\}$ holds for all $x \in M_n(X)$ and all $y \in M_k(X)$.

E, F be vector spaces. For a given mapping $h : E \to F$ and a given positive integer n, define $h_n : M_n(E) \to M_n(F)$ by, $h_n([x_{ij}]) = [h(x_{ij})]$, for all $[x_{ij}] \in M_n(E)$.

We introduce the concept of a matrix fuzzy normed space.

Definition 6. (see [13]) Let (X, N) be a fuzzy normed space.

1. $(X, \{N_n\})$ is called a matrix fuzzy normed space if for each positive integer n, $(M_n(X), N_n)$ is a fuzzy normed space and

$$N_k(AxB, t) \geq N_n\left(x, \frac{t}{\|A\| \, . \, \|B\|}\right),$$

for all $t > 0$, $A \in M_{k,n}(\mathbb{R})$, $B \in M_{n,k}(\mathbb{R})$ and $x = [x_{ij}] \in M_n(X)$ with $\|A\| \, . \, \|B\| \neq 0$.

2. $(X, \{N_n\})$ is called a matrix fuzzy Banach space if (X, N) is a fuzzy Banach space and $(X, \{N_n\})$ is a matrix fuzzy normed space.

The abstract characterization given for linear spaces of bounded Hilbert space operators in terms of matricially normed spaces implies that quotients,

mapping spaces, and various tensor products of operator spaces may again be regarded as operator spaces. Owing in part to this result, the theory of operator spaces is having an increasingly significant effect on operator algebra theory[13].

The proof given in appealed to the theory of ordered operator spaces. Effros and Ruan showed that one can give a purely metric proof of this important theorem by using a technique of Pisier and Haagerup [13].

Throughout this paper, let $(X, \|.\|_n)$ be a matrix normed space, $(Y, \|.\|_n)$ be a matrix Banach space and let n be a fixed positive integer. Let (X, N_n) be a matrix fuzzy normed space and let (Y, N_n) be a matrix fuzzy Banach space.

2. Hyers-Ulam Stability of the Cubic Functional Equation in Matrix Fuzzy Normed Spaces

In this section, we prove the generalized Hyers-Ulam stability of the generalized cubic functional equation in matrix fuzzy normed spaces using fixed point method.

Lemma 7. *(see [13]) Let $(X, \{N_n\})$ be a matrix fuzzy normed space.*

1. *$N_n(E_{kl} \otimes x, t) = N(x, t)$ for all $t > 0$ and $x \in X$*
2. *For all $[x_{ij}] \in M_n(X)$ and $t = \sum_{i,j=1}^{n} t_{ij}$,*

$$N(x_{kl}, t) \geq N_n([x_{ij}], t) \geq \min\{N(x_{ij}, t_{ij}) : i, j = 1, 2, ..., n\},$$
$$N(x_{kl}, t) \geq N_n([x_{ij}], t]) \geq \min\left\{N(x_{ij}, \frac{t}{n^2}) : i, j = 1, 2, ..., n\right\},$$

3. *$\lim_{n\to\infty} x_n = x$ if and only if $\lim_{n\to\infty} x_{ijn} = x_{ij}$ for $x_n = [x_{ijn}]$, $x = [x_{ij}] \in M_k(X)$.*

Proof. (1) since $E_{kl} \otimes x = e_k^* x e_l$ and $\|e_k^*\| = \|e_l\| = 1$, $N_n(E_{kl} \otimes x, t) \geq N(x, t)$. since $e_k(E_{kl} \otimes x)e_l^* = x$, $N_n(E_{kl} \otimes x, t) \leq N(x, t)$. So, $N(E_{kl} \otimes x, t) = N(x, t)$.

(2) $N(x_{kl}, t) = N(e_k[x_{ij}]e_l^*, t) \geq N_n([x_{ij}], \frac{t}{\|e_k\|.\|e_l\|}) = N_n([x_{ij}], t)$.

$$N_n([x_{ij}], t) = N_n\left(\sum_{i,j=1}^{n} E_{ij} \otimes x_{ij}, t\right) \geq \min\{N_n(E_{ij} \otimes x_{ij}, t_{ij}) : i, j = 1, 2, ..., n\}$$

$$= \min\{N(x_{ij}, t_{ij}) : i, j = 1, 2, ..., n\}.$$

where $t = \sum_{i,j=1}^{n} t_{ij}$. So, $N_n([x_{ij}], t) \geq \min\left\{N(x_{ij}, \frac{t}{n^2}) : i, j = 1, 2, ..., n\right\}$.

(3) By $N(x_{kl}, t) \geq N_n([x_{ij}], t) \geq \min\left\{N(x_{ij}, \frac{t}{n^2}) : i, j = 1, ..., n\right\}$, we obtain the result. □

For a mapping $f : X \to Y$, define $Df : X^2 \to Y$ and $Df_n : M_n(X^2) \to M_n(Y)$ by [12]

$$Df(x+y) = f(mx+y)+f(mx-y)-mf(x+y)-mf(x-y)-2(m^3-m)f(x),$$

$$Df_n([x_{ij}], [y_{ij}]) = f_n(m[x_{ij}] + [y_{ij}]) + f_n(m[x_{ij}] - [y_{ij}]) - mf_n([x_{ij}] + [y_{ij}]) - mf_n([x_{ij}] - [y_{ij}]) - 2(m^3 - m)f_n([x_{ij}]),$$

for all $x, y \in X$ and all $x = [x_{ij}], y = [y_{ij}] \in M_n(X)$.

Theorem 8. *Let $\phi : X^2 \to [0, \infty)$ be a function such that there exist an $\beta < 3$ with*

$$\phi(x, y) \leq \frac{\beta}{m^3}\phi\ (mx, my), \tag{1}$$

for all $x, y \in X$. Let $f : X \to Y$ be a mapping satisfying

$$N_n(Df_n([x_{ij}], [y_{ij}]), t) \geq \frac{t}{t + \sum_{i,j=1}^{n} \phi(x_{ij}, y_{ij})}, \tag{2}$$

for all $t > 0$ and $x = [x_{ij}], y = [y_{ij}] \in M_n(X)$. Then $C(x) = N - \lim_{k\to\infty} m^{3k} f(\frac{x}{m^k})$ exist for each $x \in X$ and defines a cubic mapping $C : X \to Y$ such that

$$N(f_n([x_{ij}]) - C_n([x_{ij}]), t) \geq \frac{2m^3(1-\beta)t}{2m^3(1-\beta)t + n^2\beta \sum_{i,j=1}^{n} \phi(x_{ij}, 0)}, \tag{3}$$

for all $t > 0$ and $x = [x_{ij}] \in M_n(X)$.

Proof. Let $n = 1$. Then (2) is equivalent to

$$N(f(mx + y) + f(mx - y) - mf(x + y) - mf(x - y) - 2(m^3 - m)f(x), t)$$

$$\geq \frac{t}{t+\phi(x,y)} \quad (4)$$

for all $t>0$ and $x,y\in X$.

Letting $y=0$ in (4), we get

$$N(f(mx)-m^3f(x),\frac{t}{2})\geq \frac{t}{t+\phi(x,0)}. \quad (5)$$

Let $x=\frac{x}{m}$ in (5), we get

$$N(f(x)-m^3f(\frac{x}{m}),t)\geq \frac{t}{t+\frac{1}{2}\phi(\frac{x}{m},0)}\geq \frac{t}{t+\frac{\beta}{2m^3}\phi(x,0)}$$
$$\text{for all } \ t>0 \ \text{ and } \ x\in X. \quad (6)$$

Consider the set $M=\{f:X\to Y\}$ and introduce the generalized metric on M

$$d(f,g)=\inf\left\{\mu\in\mathbb{R}_+ : N(f(x)-g(x),\mu t)\geq \frac{t}{t+\phi(x,0)},\forall x\in X,\forall t>0\right\}$$

where, as usual, $\inf\phi=+\infty$. It is easy to show that (M,d) is complete (see the proof of [11], Lemma 2.1). Now we consider the linear mapping $L:M\to M$ such that $Lf(x)=m^3f(\frac{x}{m})$, for all $x\in X$.

Let $f,g\in M$ be given such that $d(f,g)=\nu$. Then

$$N(f(x)-g(x),\nu t)\geq \frac{t}{t+\phi(x,0)},$$

for all $x\in X$, for all $t>0$. Hence

$$\begin{aligned}N(Lf(x)-Lg(x),\beta\nu t)&=N(m^3f(\frac{x}{m})-m^3g(\frac{x}{m}),\beta\nu t)\\&=N(f(\frac{x}{m})-g(\frac{x}{m}),\frac{\beta}{m^3}\nu t)\\&\geq \frac{\frac{\beta t}{m^3}}{\frac{\beta t}{m^3}+\phi(\frac{x}{m},0)}\geq \frac{\frac{\beta t}{m^3}}{\frac{\beta t}{m^3}+\frac{\beta}{m^3}\phi(x,0)}\\&=\frac{t}{t+\phi(x,0)},\end{aligned}$$

for all $x\in X$ and $t>0$. So, $d(f,g)=\nu$ implies that $d(Lf,Lg)\leq\beta\nu$.

This means that $d(Lf,Lg)\leq\beta d(f,g)$, for all $f,g\in M$.

It follows from(6) that $d(f,Lf)\leq\dfrac{\beta}{2.m^3}$.

By Theorem 2, there exist a mapping $C : X \to Y$ satisfying the following:

(1) C is a fixed point of L,i.e., $C(\frac{x}{m}) = \frac{1}{m^3}C(x)$ for all $x \in X$. The mapping C is a unique fixed point of L in the set $S = \{g \in M : d(f, g) < \infty\}$.

(2) $d(L^k f, C) \to 0$ as $k \to \infty$. This implies the equality

$$N - \lim_{k\to\infty} m^{3k} f(\frac{x}{m^k}) = C(x),$$

for all $x \in X$.

(3) $d(f, C) \leq \frac{1}{1-\beta}\, d(f, Lf)$, which implies the inequality

$$d(f, C) \leq \frac{\beta}{2m^3(1-\beta)}. \tag{7}$$

By (4),

$$N\left(m^{3k} f(\frac{mx+y}{m^k}) + m^{3k} f(\frac{mx-y}{m^k}) - m^{3k} f(\frac{x+y}{m^k}) \right.$$
$$\left. - (m^{3k} f(\frac{x-y}{m^k})) - m^{3k}(2m^3 - 2m) f(\frac{x}{m^k}), m^{3k} t\right) \geq \frac{t}{t + \phi(\frac{x}{m^k}, \frac{y}{m^k})},$$

for all $x, y \in X$ and $t > 0$. So,

$$N\left(m^{3k} f(\frac{mx+y}{m^k}) + m^{3k} f(\frac{mx-y}{m^k}) - m^{3k} f(\frac{x+y}{m^k}) \right.$$
$$\left. - (m^{3k} f(\frac{x-y}{m^k})) - m^{3k}(2m^3 - 2m) f(\frac{x}{m^k}), t\right) \geq \frac{\frac{t}{m^k}}{\frac{t}{m^k} + \frac{\beta^k}{m^{3k}}\phi(x, y)},$$

for all $x, y \in X$ and $t > 0$.

Since $\lim_{k\to\infty} \frac{\frac{t}{m^k}}{\frac{t}{m^k} + \frac{\beta^k}{m^{3k}}\phi(x,y)} = 1$, for all $x, y \in X$ and $t > 0$,

$$N\left(C(mx+y) + C(mx-y) - mC(x+y) - mC(x-y) - 2(m^3 - m)C(x), t\right) = 1,$$

for all $x, y \in X$ and $t > 0$.

Thus

$$C(mx+y)+C(mx-y)-mC(x+y)-mC(x-y)-2(m^3-m)C(x)=0.$$

So, the mapping $C:X\to Y$ is cubic. By Lemma 7 and (7),

$$\begin{aligned}N_n(f_n([x_{ij}])-C_n([x_{ij}]),t) &\geq \min\left\{N(f(x_{ij})-C(x_{ij}),\frac{t}{n^2}):i,j=1,2,...,n\right\}\\ &\geq \min\left\{\frac{2m^3(1-\beta)t}{2m^3(1-\beta)t+n^2\beta\phi(x_{ij},0)}:i,j=1,2,...,n\right\}\\ &\geq \frac{2m^3(1-\beta)t}{2m^3(1-\beta)t+n^2\beta\sum\limits_{i,j=1}^{n}\phi(x_{ij},0)} \quad \text{for all} \quad x=[x_{ij}]\in M_n(x).\end{aligned}$$

Thus $C:X\to Y$ is a unique cubic mapping satisfying (3). □

Corollary 1. *Let p,ϵ be a positive real numbers with $p<3$. Let $f:X\to Y$ be a mapping satisfying*

$$N_n(Df_n([x_{ij}],[y_{ij}]),t)\geq \frac{t}{t+\sum\limits_{i,j=1}^{n}\epsilon(\|x_{ij}\|^p+\|y_{ij}\|^p)} \tag{8}$$

for all $t>0$ and $x=[x_{ij}],y=[y_{ij}]\in M_n(X)$. Then $C(x)=N-\lim\limits_{k\to\infty}m^{3k}f(\frac{x}{m^k})$ exist for each $x\in X$ and defines a cubic mapping $C:X\to Y$ such that

$$N(f_n([x_{ij}])-C_n([x_{ij}]),t)\geq \frac{2m^3(1-m^{p-3})t}{2m^3(1-m^{p-3})t+n^2m^{p-3}\sum\limits_{i,j=1}^{n}\epsilon\|x_{ij}\|^p},$$

for all $t>0$ and $x=[x_{ij}]\in M_n(X)$.

Proof. The proof follows from Theorem 8 by taking $\phi(x,y)=\epsilon(\|x\|^p+\|y\|^p)$ for all $x,y\in X$. Then we can choose $\beta=m^{p-3}$, and we get the desired result. □

Theorem 9. *let $f:X\to Y$ be a mapping satisfying (2) for which there exist a function $\phi:X^2\to[0,\infty)$ such that there exist an $\beta<3$ with*

$$\phi(x,y)\leq m^3\beta\phi(\frac{x}{m},\frac{y}{m}),$$

for all $x, y \in X$. Then $C(x) = N - \lim_{k\to\infty} \frac{1}{m^{3k}} f(m^k x)$ exist for each $x \in X$ and defines a cubic mapping $C : X \to Y$ such that

$$N(f_n([x_{ij}]) - C_n([x_{ij}]), t) \geq \frac{2m^3 (1-\beta)\, t}{2m^3 (1-\beta)\, t + n^2 \sum_{i,j=1}^{n} \phi(x_{ij}, 0)},$$

for all $t > 0$ and $x = [x_{ij}] \in M_n(X)$.

Proof. Let (M, d) be the generalized metric space defined in the proof of Theorem 8. Now we consider the linear mapping $L : M \to M$ such that

$$Lf(x) = \frac{1}{m^3} f(mx),$$

for all $x \in X$.

It follows from (5) that $d(f, Lf) \leq \frac{1}{2.m^3}$. So

$$d(f, C) \leq \frac{1}{2m^3 (1-\beta)}.$$

The rest of the proof is similar to the proof of Theorem 8. □

Corollary 2. *Let p, ϵ be a positive real numbers with $p > 3$. Let $f : X \to Y$ be a mapping satisfying (8). Then $C(x) = N - \lim_{k\to\infty} \frac{1}{m^{3k}} f(m^k x)$ exist for each $x \in X$ and defines a cubic mapping $C : X \to Y$ such that*

$$N(f_n([x_{ij}]) - C_n([x_{ij}]), t) \geq \frac{2m^3(m^{p-3} - 1)t}{2m^3(m^{p-3} - 1)t + n^2 m^{p-3} \sum_{i,j=1}^{n} \epsilon \, \|x_{ij}\|^p},$$

for all $t > 0$ and $x = [x_{ij}] \in M_n(X)$.

Proof. The proof follows from Theorem 9 by taking $\phi(x, y) = \epsilon(\|x\|^p + \|y\|^p)$ for all $x, y \in X$. Then we can choose $\beta = m^{3-p}$, and we get the desired result. □

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