

**EXISTENCE OF SOLUTIONS FOR NONLINEAR FRACTIONAL
IMPULSIVE STOCHASTIC DIFFERENTIAL EQUATIONS
WITH NONLOCAL CONDITIONS AND INFINITE DELAY**

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Abstract: This paper is concerned with the existence of mild solutions for fractional stochastic impulsive functional infinite delay integrodifferential systems with nonlocal initial conditions in Hilbert spaces. Using fractional calculations, fixed point technique, stochastic analysis theory and methods adopted directly from deterministic fractional equations, new set of sufficient conditions are formulated and proved for the existence of mild solutions for the fractional impulsive stochastic differential equation with nonlocal conditions and infinite delay.

AMS Subject Classification: 65C30, 34G20, 34K45

Key Words: fractional impulsive stochastic differential equations, Fixed point principle, mild solution, nonlocal condition

Received: January 16, 2014

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url: www.acadpubl.eu

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1. Introduction

Stochastic differential equations is an emerging field and has attracted great interest from both theoretical and applied disciplines, which has been successfully applied to problems in physics, biology, chemistry, mechanics and so on. (see [8, 9, 11, 18, 20]) and the references therein). In the present literature, there are many papers on the existence and uniqueness of solutions to stochastic differential equations (see [3, 4, 24]).

The stochastic differential equations with infinite delay have become important in recent years as mathematical models of phenomena in both physical and social sciences [25, 26]. Wei and Wang [26] considered a class of stochastic functional differential equations with infinite delay in which the initial value belongs to the phase space.

However, only few papers deal with the existence result for stochastic fractional systems [24]. Cui and Yan [13] studied the existence of mild solutions for a class of fractional neutral stochastic integrodifferential equations with infinite delay in Hilbert spaces by means of Sadovskii's fixed point theorem. The existence and uniqueness for a class of fractional stochastic delay and evolution differential equations has been established in [14, 15].

The problem with nonlocal condition, which is a generalization of the problem of classical condition, was motivated by physical problems. The pioneering work on nonlocal conditions is due to Byszewski (see [5, 6, 7]). Since it is demonstrated that the nonlocal problems have better effects in applications than the classical Cauchy problems. Stochastic differential equations with nonlocal conditions were studied by many authors and some basic results on nonlocal problems have been obtained, see [2, 24] and the references contained therein.

More recently, Sakthivel and al. [24] studies the existence of mild solutions for a class of fractional stochastic differential equations with impulses in Hilbert spaces. The existence and uniqueness of mild solutions for semilinear stochastic fractional differential equations with nonlocal conditions, finite delay and without impulse has been derived.

For our best knowledge, there is no work reported on fractional impulsive stochastic differential equation with nonlocal conditions and infinite delay. Motivated by the above works, the purpose of this paper is to prove the existence and uniqueness of mild solutions for the fractional impulsive stochastic differential equation with nonlocal conditions and infinite delay. Our approach is based on the fixed point theorem. The rest of this paper is organized as follows. In Section 2, impulsive stochastic differential equations are presented,

together with definition of mild solution. Finally in Section 3, the existence and uniqueness results on mild solutions are derived.

2. Preliminaries and Basic Properties

Let $\mathbb{H}, \mathbb{K}$ be two separable Hilbert spaces and $L(\mathbb{K}, \mathbb{H})$ be the space of bounded linear operators from $\mathbb{K}$ into $\mathbb{H}$. For convenience, we will use the same notation $\| \cdot \|$ to denote the norms in $\mathbb{H}, \mathbb{K}$ and $L(\mathbb{K}, \mathbb{H})$, and use $(\cdot, \cdot)$ to denote the inner product of $\mathbb{H}$ and $\mathbb{K}$ without any confusion. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ be a complete filtered probability space satisfying that $\mathcal{F}_0$ contains all $\mathbb{P}$ -null sets of $\mathcal{F}$. $W = (W_t)_{t \geq 0}$ be a Q -Wiener process defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ with the covariance operator Q such that $TrQ < \infty$. We assume that there exists a complete orthonormal system $\{e_k\}_{k \geq 1}$ in $\mathbb{K}$, a bounded sequence of nonnegative real numbers λ_k such that $Qe_k = \lambda_k e_k$, $k = 1, 2, \dots$, and a sequence of independent Brownian motions $\{\beta_k\}_{k \geq 1}$ such that

$$(\omega(t), e)_{\mathbb{K}} = \sum_{k=1}^{\infty} \sqrt{\lambda_k} (e_k, e)_{\mathbb{K}} \beta_k(t), \quad e \in \mathbb{K}, \quad t \geq 0$$

Let $\mathcal{L}_2^0 = \mathcal{L}_2(\mathbb{Q}^{\frac{1}{2}} \mathbb{K}, \mathbb{H})$ be the space of all HilbertSchmidt operators from $\mathbb{Q}^{\frac{1}{2}} \mathbb{K}$ to $\mathbb{H}$ with the inner product $\langle \varphi, \psi \rangle_{\mathcal{L}_2^0} = Tr[\varphi Q \psi^*]$

we consider the following fractional stochastic impulsive integrodifferential systems with nonlocal conditions :

$$\begin{cases} D_t^\alpha x(t) = Ax(t) + f(t, x_t, B_1 x(t)) \\ + \sigma(t, x_t, B_2 x(t)) \frac{d\omega(t)}{dt}, t \in J = [0, T], T > 0, t \neq t_k \\ \Delta x(t_k) = I_k(x(t_k^-)), \quad k = 1, \dots, m \\ x(0) + g(x) = x_0 = \phi, \quad \phi \in B_h. \end{cases} \quad (1)$$

where D_t^α is the Caputo fractional derivative of order α , $0 < \alpha < 1$, $x(\cdot)$ takes the value in the separable Hilbert space $\mathbb{H}$; $A : D(A) \in \mathbb{H} \rightarrow \mathbb{H}$ is the infinitesimal generator of an α -resolvent family $S_\alpha(t)_{t \geq 0}$. The history $x_t : (-\infty, 0] \rightarrow \mathbb{H}, x_t(\theta) = x(t + \theta)$, $\theta \leq 0$ belongs to an abstract phase space $\mathcal{B}_h$; $f : J \times \mathcal{B}_h \times \mathbb{H} \rightarrow \mathbb{H}$, $\sigma : J \times \mathcal{B}_h \times \mathbb{H} \rightarrow \mathcal{L}_2^0$ and $g : B_h \rightarrow \mathbb{H}$ are appropriate functions to be specified later; $I_k : \mathbb{H} \rightarrow \mathbb{H}$, $(k = 1, 2, \dots, m)$, are appropriate functions. The terms $B_1 x(t)$ and $B_2 x(t)$ are given by $B_1 x(t) = \int_0^t K(t, s)x(s)ds$ and $B_2 x(t) = \int_0^t P(t, s)x(s)ds$ respectively, where $K, P \in C(D, \mathbb{R}^+)$ are the set of all positive continuous functions on $D = \{(t, s) \in \mathbb{R}^2 : 0 \leq s \leq t \leq T\}$. Here

$0 = t_0 \leq t_1 \leq \dots \leq t_m \leq t_{m+1} = T$, $\Delta x(t_k) = I_k(x(t_k^-)) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{h \rightarrow 0} x(t_k + h)$ and $x(t_k^-) = \lim_{h \rightarrow 0} x(t_k - h)$ represent the right and left limits of $x(t)$ at $t = t_k$ respectively. The initial data $\phi = \{\phi(t); t \in (-\infty, 0]\}$ is an $\mathcal{F}_0$ -measurable, $\mathcal{B}_h$ -valued random variable independent of ω with finite second moments.

Now, we present the abstract space phase $\mathcal{B}_h$. Assume that $h : (-\infty, 0] \rightarrow (0, +\infty)$ with $l = \int_{-\infty}^0 h(t)dt < \infty$ a continuous function. We define the abstract phase space $\mathcal{B}_h$ by

$$\mathcal{B}_h := \left\{ \phi : (-\infty, 0] \rightarrow \mathbb{H}, \text{ for any } a > 0, (E | \phi(\theta)^2)^{\frac{1}{2}} \right. \\ \left. \text{is bounded and measurable function on } [-a, 0] \text{ and } \int_{-\infty}^0 h(s) \sup_{s \leq \theta \leq 0} (E | \phi(\theta)^2)^{\frac{1}{2}} < +\infty. \right\}$$

If $\mathcal{B}_h$ is endowed with the norm

$$\|\phi\|_{\mathcal{B}_h} := \int_{-\infty}^0 h(s) \sup_{s \leq \theta \leq 0} (E | \phi(\theta)^2)^{\frac{1}{2}}, \phi \in \mathcal{B}_h,$$

then $(\mathcal{B}_h, \|\cdot\|_{\mathcal{B}_h})$ is a Banach space [21, 22].

Now we consider the space

$$\mathcal{B}_b := \{x : (-\infty, T] \rightarrow \mathbb{H}, \text{ such that } x|_{J_k} \in C(J_k, \mathbb{H}) \\ \text{and there exist } x(t_k^+), \text{ and } x(t_k^-) \\ \text{with } x(t_k) = x(t_k^-), x_0 = \phi \in \mathcal{B}_h, k = 1, \dots, m\}$$

where $x|_{J_k}$ is the restriction of x to $J_k = (t_k, t_{k+1}]$, $k = 0, 1, 2, \dots, m$. We endow a seminorm $\|\cdot\|_{\mathcal{B}_b}$ on $\mathcal{B}_b$, it is defined by

$$\|x\|_{\mathcal{B}_b} = \|\phi\|_{\mathcal{B}_h} + \sup_{0 \leq s \leq T} (E \|x(s)\|^2)^{\frac{1}{2}}, x \in \mathcal{B}_b.$$

We recall the following lemma

Lemma 1. [22] Assume that $x \in \mathcal{B}_b$; then for $t \in J$, $x_t \in \mathcal{B}_h$. Moreover

$$l(E \|x(t)\|^2)^{\frac{1}{2}} \leq l \sup_{s \in [0, t]} E \|x(s)\|^2)^{\frac{1}{2}} + \|x_0\|_{\mathcal{B}_h}$$

where $l = \int_{-\infty}^0 h(s)ds < \infty$.

Definition 2. A two parameter function of the Mittag-Leffler type is defined by the series expansion

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} = \frac{1}{2\pi i} \int_C \frac{\mu^{\alpha-\beta} e^{\mu}}{\mu^{\alpha} - z} d\mu, \quad \alpha, \beta \in C, R(\alpha) > 0 \quad (2)$$

where C is a contour which starts and ends at $-\infty$ and encircles the disc $|\mu| \leq |z|^{\frac{1}{\alpha}}$ counter clockwise.

For short, $E_{\alpha}(z) = E_{\alpha,1}(z)$. It is an entire function which provides a simple generalization of the exponent function: $E_1(z) = e^z$ and the cosine function: $E_2(z^2) = \cosh(z)$, $E_2(-z^2) = \cos(z)$, and plays a vital role in the theory of fractional differential equations. The most interesting properties of the Mittag-Leffler functions are associated with their Laplace integral

$$\int_0^{\infty} e^{-\lambda t} t^{\beta-1} E_{\alpha,\beta}(\omega t^{\alpha}) dt = \frac{\lambda^{\alpha-\beta}}{\lambda^{\alpha} - \omega}, \quad Re\lambda > \omega^{\frac{1}{\alpha}}, \omega > 0.$$

and for more details see [19].

Definition 3. [17] A closed and linear operator A is said to be sectorial if there are constants $\omega \in \mathbb{R}$, $\theta \in [\frac{\pi}{2}, \pi]$, $M > 0$ such that the following two conditions are satisfied:

$$(1) \rho(A) \subset \sum_{(\theta,\omega)} = \{\lambda \in C : \lambda \neq \omega, |\arg(\lambda - \omega)| < \theta\},$$

$$(2) \|\mathbb{R}(\lambda, A)\| \leq \frac{M}{|\lambda\omega|}, \lambda \in \sum_{(\theta,\omega)}$$

Definition 4. [1] Let A be a closed and linear operator with the domain $D(A)$ defined in a Banach space X . Let $\rho(A)$ be the resolvent set of A . We say that A is the generator of an α -resolvent family if there exist $\omega \geq 0$ and a strongly continuous function $S_{\alpha} : \mathbb{R}_+ \rightarrow \mathcal{L}(X)$, where $\mathcal{L}(X)$ is a Banach space of all bounded linear operators from X into X and the corresponding norm is denoted by $\|\cdot\|$, such that $\{\lambda^{\alpha} : Re\lambda > \omega\} \subset \rho(A)$ and

$$(\lambda^{\alpha} I - A)^{-1} x = \int_0^{\infty} e^{\lambda t} S_{\alpha}(t) x dt, \quad Re\lambda > \omega, x \in X \quad (3)$$

where $S_{\alpha}(t)$ is called the α -resolvent family generated by A .

Definition 5. [16] Let A be a closed linear operator with the domain $D(A)$ defined in a Banach space X and $\alpha > 0$. We say that A is the generator

of a solution operator if there exist $\omega \geq 0$ and a strongly continuous function $S_\alpha : \mathbb{R}_+ \rightarrow \mathcal{L}(X)$ such that $\{\lambda\alpha : \operatorname{Re} \lambda > \omega\} \subset \rho(A)$ and

$$\lambda^{\alpha-1}(\lambda^\alpha I - A)^{-1}x = \int_0^\infty e^{\lambda t} S_\alpha(t)x dt, \quad \operatorname{Re} \lambda > \omega, x \in X \quad (4)$$

where $S_\alpha(t)$ is called the solution operator generated by A .

The concept of the solution operator is closely related to the concept of a resolvent family [16]. For more details on α -resolvent family and solution operators, we refer the reader to [12, 16] and the references therein.

Definition 6. [10] The Caputo derivative of order α for a function $f : [0, \infty) \rightarrow \mathbb{R}$, which is at least n -times differentiable can be defined as

$$D_a^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-s)^{n-\alpha-1} f^{(n)}(s) ds = I_a^{(n-\alpha)} \left(\frac{d^n f}{dt^n} \right) (t) \quad (5)$$

for $n-1 \leq \alpha < n, n \in \mathbb{N}$. If $0 < \alpha \leq 1$, then

$$D_a^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t (t-s)^{-\alpha} \left(\frac{df(s)}{ds} \right) ds \quad (6)$$

Obviously, the Caputo derivative of a constant is equal to zero. The Laplace transform of the Caputo derivative of order $\alpha > 0$ is given as

$$L\{D_t^\alpha f(t); \lambda\} = \lambda^\alpha \hat{f}(\lambda) - \sum_{k=0}^{n-1} \lambda^{\alpha-k-1} f^{(k)}(0) \quad n-1 \leq \alpha < n.$$

Definition 7. The fractional integral of order α with the lower limit 0 for a function f is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (s-t)^{\alpha-1} f(s) ds \quad (7)$$

provided the right-hand side is pointwise defined on $[0, \infty)$, where the Γ is the gamma function.

Lemma 8. [16] If f satisfies the uniform Holder condition with the exponent $\beta \in (0, 1]$ and A is a sectorial operator, then the unique solution of the Cauchy problem

$$D_t^\alpha x(t) = Ax(t) + f(t, x_t, Bx(t)), \quad t > t_0, t_0 \geq 0, 0 < \alpha < 1 \quad (8)$$

$$x(t) = \phi(t), \quad t \leq t_0, \quad (9)$$

is given by

$$x(t) = T_\alpha(t - t_0)(x(t_0^+)) + \int_{t_0}^t S_\alpha(t - s)f(s, x_s, Bx(s))ds, \quad (10)$$

where

$$T_\alpha(t) = E_{\alpha,1}(At^\alpha) = \frac{1}{2\pi i} \int_{\widehat{B}_r} e^{\lambda t} \frac{\lambda^{\alpha-1}}{\lambda^\alpha - A} d\lambda \quad (11)$$

$$S_\alpha(t) = t^{\alpha-1} E_{\alpha,\alpha}(At^\alpha) = \frac{1}{2\pi i} \int_{\widehat{B}_r} e^{\lambda t} \frac{1}{\lambda^\alpha - A} d\lambda \quad (12)$$

here $\widehat{B}_r$ denotes the Bromwich path; $S_\alpha(t)$ is called the α -resolvent family and $T_\alpha(t)$ is the solution operator generated by A .

Next, we mention the statement of Krasnoselskii's fixed point theorem [16].

Theorem 9. *Let B be a nonempty closed convex of a Banach space $(X, \|\cdot\|)$. Suppose that P and Q map B into X such that*

(i) $Px + Qy \in B$ whenever $x, y \in B$;

(ii) P is compact and continuous;

(iii) Q is a contraction mapping.

Then there exists $z \in B$ such that $z = Pz + Qz$.

Definition 10. An $\mathcal{F}_t$ -adapted stochastic process $x : (-\infty, T] \rightarrow \mathbb{H}$ is called a solution of the system (1) if $x(0) + g(x) = \phi \in \mathcal{B}_h$ the impulsive condition $\Delta x(t_k) = I_k(x(t_k^-))$, $k = 1, \dots, m$, is verified, the restriction of $x(\cdot)$ to the interval J_k ($k = 0, 1, \dots, m$) is continuous, and the following integral equation holds:

$$\begin{aligned} x(t) = & T_\alpha(t)(\phi(0) - g(x)) + \int_0^t S_\alpha(t - s)f(s, x_s, B_1x(s))ds \\ & + \int_0^t S_\alpha(t - s)\sigma(s, x_s, B_2x(s))d\omega(t) + \sum_{0 < t_k < t} T_\alpha(t - t_k)I_k(x(t_k^-)). \end{aligned} \quad (13)$$

3. Main Results

In this section, we first establish the existence of solutions to nonlinear fractional stochastic equation (1). More precisely, we will formulate and prove sufficient conditions for the existence of solutions to (1) with nonlocal condition and infinite delay and impulses.

For $\phi \in \mathcal{B}_h$, we define $\widehat{\phi}$ by

$$\widehat{\phi}(t) = \begin{cases} \phi(t), & t \in (-\infty, 0], \\ T(t)\phi(0), & t \in J; \end{cases} \quad \text{then } \widehat{\phi} \in \mathcal{B}_b.$$

Let $x(t) = y(t) + \widehat{\phi}(t)$, $-\infty < t < T$

It is evident that y satisfies $y_0 = 0$, $t \in (-\infty, 0]$, and

$$\begin{aligned} y(t) = & T_\alpha(t)(-g(y + \widehat{\phi})) + \int_0^t S_\alpha(t-s)f(s, y_s + \widehat{\phi}_s, B_1(y(s) + \widehat{\phi}(s)))ds \\ & + \int_0^t S_\alpha(t-s)\sigma(s, y_s + \widehat{\phi}_s, B_2(y(s) + \widehat{\phi}(s)))d\omega(t) \\ & + \sum_{0 < t_k < t} T_\alpha(t-t_k)I_k(y(t_k^-) + \widehat{\phi}(t_k^-)). \end{aligned}$$

if and only if x satisfies $x(t) = \phi(t)$, $t \in (-\infty, 0]$, and

$$\begin{aligned} x(t) = & T_\alpha(t)(\phi(0) - g(x)) + \int_0^t S_\alpha(t-s)f(s, x_s, B_1x(s))ds \\ & + \int_0^t S_\alpha(t-s)\sigma(s, x_s, B_2x(s))d\omega(t) \\ & + \sum_{0 < t_k < t} T_\alpha(t-t_k)I_k(x(t_k^-)), t \in J \end{aligned}$$

In order to establish the results, we impose the following conditions

(H1) If $\alpha \in (0, 1)$ and $A \in \mathbb{A}^\alpha(\theta_0, \omega_0)$, then for any $x \in \mathbb{H}$ and $t > 0$ we have $\|T_\alpha(t)\| \leq Me^{\omega t}$ and $\|S_\alpha(t)\| \leq Ce^{\omega t}(1 + t^{\alpha-1})$, $\omega > \omega_0$. Thus, we have

$$\|T_\alpha(t)\| \leq \tilde{M}_T \quad \text{and} \quad \|S_\alpha(t)\| \leq t^{\alpha-1}\tilde{M}_S$$

where $\tilde{M}_T = \sup_{0 \leq t \leq T} \|T_\alpha(t)\|$ and $\tilde{M}_S = \sup_{0 \leq t \leq T} Ce^{\omega t}(1 + t^{\alpha-1})$ (for more details, see [25]).

(H2) There existe $\mu_1, \mu_2 > 0$ such that

$$E\|f(t, \gamma, x) - f(t, \psi, y)\|_{\mathbb{H}}^2 \leq \mu_1\|\gamma - \psi\|_{\mathcal{B}_h}^2 + \mu_2E\|x - y\|_{\mathbb{H}}^2.$$

(H3) There existe $\nu_1, \nu_2 > 0$ such that

$$E\|\sigma(t, \gamma, x) - f(t, \psi, y)\|_{\mathcal{L}_2^0}^2 \leq \nu_1 \|\gamma - \psi\|_{\mathcal{B}_h}^2 + \nu_2 E\|x - y\|_{\mathbb{H}}^2.$$

(H4) For each $k = 1, 2, \dots, m$ there exist $n_k > 0$ such that

$$E\|I_k(x) - I_k(y)\|_{\mathbb{H}}^2 \leq n_k E\|x - y\|_{\mathbb{H}}^2, \quad \text{for all } x, y \in \mathbb{H}$$

(H5) For $g : \mathcal{B}_h \rightarrow \mathbb{H}$, is continuous, and there exist some positive constants δ_1, δ_2 such that

$$E\|g(x) - g(y)\|_{\mathbb{H}}^2 \leq \delta_1 \|x - y\|_{\mathcal{B}_h}^2, \quad \text{and} \quad E\|g(x)\|_{\mathbb{H}}^2 \leq \delta_1 \|x\|_{\mathcal{B}_h}^2 + \delta_2$$

Now, we are in a position to state the existence theorem. Our first theorem is based on the Banach contraction principle.

Theorem 11. Assume that the conditions (H1) – (H4) hold. If $A \in A_\alpha(\theta_0, \omega_0)$, then the system (1) has a unique mild solution provided that

$$\begin{aligned} & \left[4\tilde{M}_T^2 \left(\delta + \sum_{1 \leq k \leq m} n_k \right) + 4\tilde{M}_S^2 T^{2\alpha} \left(\frac{1}{\alpha^2} (\mu_1 l + \mu_2 B_1^*) \right. \right. \\ & \quad \left. \left. + \frac{1}{T(2\alpha - 1)} (\nu_1 l + \nu_2 B_2^*) \right) \right] < 1 \end{aligned} \quad (14)$$

where $B_1^* = \sup_{t \in [0, T]} \int_0^t K(t, s) ds < \infty$

and $B_2^* = \sup_{t \in [0, T]} \int_0^t P(t, s) ds < \infty$.

Proof. Define $\Psi : \mathcal{B}_b \rightarrow \mathcal{B}_b$ by $\Psi y(t) = 0, \quad t \in (-\infty, 0]$,

$$\begin{aligned} \Psi y(t) = & T_\alpha(t)(-g(y + \hat{\phi})) \\ & + \int_0^t S_\alpha(t-s)f(s, y_s + \hat{\phi}_s, B_1(y(s) + \hat{\phi}(s)))ds \\ & + \int_0^t S_\alpha(t-s)\sigma(s, y_s + \hat{\phi}_s, B_2(y(s) + \hat{\phi}(s)))d\omega(t) \\ & + \sum_{0 \leq t_k < t} T_\alpha(t-t_k)I_k(y(t_k^-) + \hat{\phi}(t_k^-)), t \in J. \\ & := \sum_{i=1}^4 \Theta_i(t) \end{aligned} \quad (15)$$

Set $\mathcal{B}_b^0 = \{z \in \mathcal{B}_b, \text{ such that } z_0 = 0\}$ and for any $z \in \mathcal{B}_b^0$ we have

$$\|z\|_{\mathcal{B}_b^0} = \|z_0\|_{\mathcal{B}_h} + \sup_{t \in J} (E\|z(t)\|^2)^{\frac{1}{2}} = \sup_{t \in J} (E\|z(t)\|^2)^{\frac{1}{2}}$$

thus $(\mathcal{B}_b^0, \|\cdot\|_{\mathcal{B}_b^0})$ is a Banach space.

Let $y, y^* \in \mathcal{B}_b^0$ then for all $t \in J$

$$\begin{aligned}
& E\|(\Psi y)(t) - (\Psi y^*)(t)\|_{\mathbb{H}}^2 \leq 4\|T_\alpha(t)\|^2 E\|g(y) - g(y^*)\|_{\mathbb{H}}^2 \\
& + 4\sum_{0 < t_k < t} \|T_\alpha(t - t_k)\|^2 E\|I_k y(t_k^-) - I_k y^*(t_k^-)\|_{\mathbb{H}}^2 \\
& + 4E\left\|\int_0^t S_\alpha(t-s) \left[f(s, y_s + \widehat{\phi}_s, B_1(y(s) + \widehat{\phi}(s))) \right. \right. \\
& \quad \left. \left. - f(s, y_s^* + \widehat{\phi}_s, B_1(y^*(s) + \widehat{\phi}(s))) \right] ds\right\|_{\mathbb{H}}^2 \\
& + 4E\left\|\int_0^t S_\alpha(t-s) \left[\sigma(s, y_s + \widehat{\phi}_s, B_2(y(s) + \widehat{\phi}(s))) \right. \right. \\
& \quad \left. \left. - \sigma(s, y_s^* + \widehat{\phi}_s, B_2(y^*(s) + \widehat{\phi}(s))) \right] d\omega(t)\right\|_{\mathbb{H}}^2 \\
& \leq 4\tilde{M}_T^2 \left[E\|g(y) - g(y^*)\|_{\mathbb{H}}^2 + \sum_{0 < t_k < t} E\|I_k y(t_k^-) - I_k y^*(t_k^-)\|_{\mathbb{H}}^2 \right] \\
& + 4\tilde{M}_S^2 \int_0^t (t-s)^{\alpha-1} ds \int_0^t (t-s)^{\alpha-1} \left[\mu_1 \|y(s) - y^*(s)\|_{\mathcal{B}_h}^2 \right. \\
& \quad \left. + \mu_2 E\|B_1(y(s) + \widehat{\phi}(s)) - B_1(y^*(s) + \widehat{\phi}(s))\|_{\mathbb{H}}^2 \right] ds \\
& + 4\tilde{M}_S^2 \int_0^t (t-s)^{2(\alpha-1)} \left[\nu_1 \|y_s - y_s^*\|_{\mathcal{B}_h}^2 \right. \\
& \quad \left. + \nu_2 E\|B_2(y(s) + \widehat{\phi}(s)) - B_2(y^*(s) + \widehat{\phi}(s))\|_{\mathbb{H}}^2 \right] ds \\
& \leq 4\tilde{M}_T^2 (\delta_1 + \sum_{1 \leq k \leq m} n_k) \sup E\|y(s) - y^*(s)\|_{\mathbb{H}}^2 \\
& + 4\tilde{M}_S^2 \frac{T^\alpha}{\alpha} \int_0^t (t-s)^{\alpha-1} [\mu_1 l \sup E\|y(s) - y^*(s)\|_{\mathbb{H}}^2 \\
& \quad + \mu_2 B_1^* \sup E\|y(s) - y^*(s)\|_{\mathbb{H}}^2] ds \\
& + 4\tilde{M}_S^2 \int_0^t (t-s)^{2(\alpha-1)} [\nu_1 l \sup E\|y(s) - y^*(s)\|_{\mathbb{H}}^2 \\
& \quad + \nu_2 B_2^* \sup E\|y(s) - y^*(s)\|_{\mathbb{H}}^2] ds \\
& \leq \left(4\tilde{M}_T^2 \left(\delta_1 + \sum_{1 \leq k \leq m} n_k \right) + 4\tilde{M}_S^2 T^{2\alpha} \left[\frac{1}{\alpha^2} (\mu_1 l + \mu_2 B_1^*) \right. \right. \\
& \quad \left. \left. + \frac{1}{T(2\alpha-1)} (\nu_1 l + \nu_2 B_2^*) \right] \right) \|y - y^*\|_{\mathcal{B}_b^0}^2
\end{aligned}$$

Hence, Ψ is a contraction map by the condition (14) and therefore it has a unique fixed point $y \in \mathcal{B}_0^2$, which is a mild solution of (1) on $(-\infty, T]$. The proof is complete. $\square$

The second result is established using Krasnoselskii's fixed point theorem. Now, we make the following assumptions.

(H6) $f : J \times \mathcal{B}_h \times \mathbb{H} \rightarrow \mathbb{H}$ is continuous and there exist two continuous functions $\mu_1, \mu_2 : J \rightarrow (0, \infty)$ such that

$$E\|f(t, \psi, x)\|_{\mathbb{H}}^2 \leq \mu_1(t) \|\psi\|_{\mathcal{B}_h}^2 + \mu_2(t) E\|x\|_{\mathbb{H}}^2, (t, \psi, x) \in J \times \mathcal{B}_h \times \mathbb{H}$$

where $\mu_1^* = \sup_{s \in [0, t]} \mu_1(s)$ and $\mu_2^* = \sup_{s \in [0, t]} \mu_2(s)$.

(H7) $\sigma : J \times \mathcal{B}_h \times \mathbb{H} \rightarrow \mathcal{L}_2^0$ is continuous and there exist two continuous functions $\nu_1, \nu_2 : J \rightarrow (0, \infty)$ such that

$$E\|\sigma(t, \psi, x)\|_{\mathcal{L}_2^0}^2 \leq \nu_1(t)\|\psi\|_{\mathcal{B}_h}^2 + \nu_2(t)E\|x\|_{\mathbb{H}}^2, (t, \psi, x) \in J \times \mathcal{B}_h \times \mathcal{L}_2^0$$

where $\nu_1^* = \sup_{s \in [0, t]} \nu_1(s)$ and $\nu_2^* = \sup_{s \in [0, t]} \nu_2(s)$.

(H8) The function $I_k : \mathbb{H} \rightarrow \mathbb{H}$ is continuous and there exist continuous non-decreasing functions L_k such that, for each $x \in \mathbb{H}$,

$$E\|I_k(x)\|_{\mathbb{H}}^2 \leq L_k E\|x\|_{\mathbb{H}}^2$$

Theorem 12. Suppose that conditions (H1) – (H4) and (H6) – (H8) are satisfied with

$$r \geq 4\tilde{M}_T^2 \lambda_3 + 4\tilde{M}_S^2 T^{2\alpha} \left[\frac{\lambda_1}{\alpha^2} + \frac{\lambda_2}{T(2\alpha - 1)} \right] \quad (16)$$

and

$$\tilde{M}_S^2 T^{2\alpha} \left[\frac{1}{\alpha^2} (\mu_1 l + \mu_2 B_1^*) + \frac{1}{T(2\alpha - 1)} (\nu_1 l + \nu_2 B_2^*) \right] < 1 \quad (17)$$

Then system (1) has a mild solution on $(-\infty, T]$

Proof. Let $\Psi_1 : \mathcal{B}_r \rightarrow \mathcal{B}_r$ and $\Psi_2 : \mathcal{B}_r \rightarrow \mathcal{B}_r$ be defined as

$$(\Psi_1 y)(t) = \begin{cases} T_\alpha(t)(-g(y + \hat{\phi})) \\ + \sum_{0 < t_k < t} T_\alpha(t - t_k) I_k(y(t_k^-) + \hat{\phi}(t_k^-)) \end{cases} \quad t \in [0, T] \quad (18)$$

and

$$(\Psi_2 y)(t) = \begin{cases} \int_0^t S_\alpha(t - s) f(s, y_s + \hat{\phi}_s, B_1(y(s) + \hat{\phi}(s))) ds \\ + \int_0^t S_\alpha(t - s) \sigma(s, y_s + \hat{\phi}_s, B_2(y(s) + \hat{\phi}(s))) d\omega(s) \end{cases} \quad t \in [0, T] \quad (19)$$

In order to use Theorem 9 we will verify that Ψ_1 is compact and continuous while Ψ_2 is a contraction operator. Set $\mathcal{B}_r = \left\{ y \in \mathcal{B}_b^0, \quad \|y\|_{\mathcal{B}_b^0}^2 \leq r, \quad r > 0 \right\}$.

The set $\mathcal{B}_r$ is clearly a bounded closed convex set in $\mathcal{B}_b^0$ for each $r > 0$ and For each $y \in \mathcal{B}_r$. by lemme 1 we have

$$\begin{aligned} \|y_t + \hat{\phi}_t\|_{\mathcal{B}_h}^2 &\leq 2(\|y_t\|_{\mathcal{B}_h}^2 + \|\hat{\phi}_t\|_{\mathcal{B}_h}^2) \\ &\leq 4(l^2 \sup_{s \in [0, t]} E\|y(s)\|_{\mathbb{H}}^2 + \|y_0\|_{\mathcal{B}_h}^2) \\ &\quad + 4(l^2 \sup_{s \in [0, t]} E\|\hat{\phi}(s)\|_{\mathbb{H}}^2 + \|\hat{\phi}_0\|_{\mathcal{B}_h}^2) \\ &\leq 4(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) \end{aligned}$$

For the sake of convenience, we divide the proof into several steps.

step 1. We show that $\Psi_1 y + \Psi_2 y^* \in \mathcal{B}_r$, for $y, y^* \in \mathcal{B}_r$. For $t \in [0, T]$, we have

$$\begin{aligned}
& \|(\Psi_1 y)(t) + (\Psi_2 y^*)(t)\|_{\mathbb{H}}^2 \leq 4\|T_\alpha(t)\|^2 E\|(-g(y + \hat{\phi}))\|_{\mathbb{H}}^2 \\
& + 4\sum_{0 < t_k < t} \|T_\alpha(t - t_k)\|^2 E\|I_k(y(t_k^-) + \hat{\phi}(t_k^-))\|_{\mathbb{H}}^2 \\
& + 4E\left\|\int_0^t S_\alpha(t-s)f(s, y_s^* + \hat{\phi}_s, B_1(y^*(s) + \hat{\phi}(s)))ds\right\|_{\mathbb{H}}^2 \\
& + 4E\left\|\int_0^t S_\alpha(t-s)\sigma(s, y_s^* + \hat{\phi}_s, B_2(y^*(s) + \hat{\phi}(s)))d\omega(t)\right\|_{\mathbb{H}}^2 \\
& \leq 4\tilde{M}_T^2(\delta_1\|y_t + \hat{\phi}_t\|_{\mathcal{B}_h}^2 + \delta_2) + 4\tilde{M}_T^2\sum_{k=1}^m L_k\|y_t + \hat{\phi}_t\|_{\mathcal{B}_h}^2 \\
& + 4\int_0^t \|S_\alpha(t-s)\|ds\int_0^t \|S_\alpha(t-s)\|E\|f(s, y_s^* \\
& + \hat{\phi}_s, B_1(y^*(s) + \hat{\phi}(s)))\|_{\mathbb{H}}^2 ds \\
& + 4\int_0^t \|S_\alpha(t-s)\|^2 E\|\sigma(s, y_s^* + \hat{\phi}_s, B_2(y^*(s) + \hat{\phi}(s)))\|_{\mathbb{H}}^2 ds \\
& \leq 4\tilde{M}_T^2(\delta_1\|y_t + \hat{\phi}_t\|_{\mathcal{B}_h}^2 + \delta_2) + 4\tilde{M}_T^2\sum_{k=1}^m L_k\|y_t + \hat{\phi}_t\|_{\mathcal{B}_h}^2 \\
& + 4\tilde{M}_S^2\int_0^t (t-s)^{\alpha-1}ds\int_0^t (t-s)^{\alpha-1}[\mu_1(s)\|y_s^* + \hat{\phi}_s\|_{\mathcal{B}_h}^2 \\
& + \mu_2(s)E\|B_1(y^*(s) + \hat{\phi}(s))\|_{\mathbb{H}}^2]ds \\
& + 4\tilde{M}_S^2\int_0^t (t-s)^{2(\alpha-1)}[\nu_1(s)\|y_s^* + \hat{\phi}_s\|_{\mathcal{B}_h}^2 \\
& + \nu_2(s)E\|B_2(y^*(s) + \hat{\phi}(s))\|_{\mathbb{H}}^2]ds \\
& \leq 4\tilde{M}_T^2\left\{\delta_2 + 4(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2))(\delta_1 + \sum_{k=1}^m L_k)\right\} \\
& + 4\tilde{M}_S^2\frac{T^\alpha}{\alpha}\int_0^t (t-s)^{\alpha-1}[4\mu_1^*(\|\phi\|_{\mathcal{B}_h}^2 \\
& + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \mu_2^* B_1^* \sup_{s \in [0, T]} E\|y^*(s)\|_{\mathbb{H}}^2]ds \\
& + 4\tilde{M}_S^2\int_0^t (t-s)^{2(\alpha-1)}[4\nu_1^*(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) \\
& + \nu_2^* B_2^* \sup_{s \in [0, T]} E\|y^*(s)\|_{\mathbb{H}}^2]ds \\
& \leq 4\tilde{M}_T^2\left\{\delta_2 + 4(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2))(\delta_1 + \sum_{k=1}^m L_k)\right\} \\
& + 4\tilde{M}_S^2\frac{T^{2\alpha}}{\alpha^2}[4\mu_1^*(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \mu_2^* B_1^* r] \\
& + 4\tilde{M}_S^2\frac{T^{2\alpha-1}}{2\alpha-1}[4\nu_1^*(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \nu_2^* B_2^* r] \\
& = 4\tilde{M}_T^2\lambda_3 + 4\tilde{M}_S^2T^{2\alpha}\left[\frac{\lambda_1}{\alpha^2} + \frac{\lambda_2}{T(2\alpha-1)}\right]
\end{aligned}$$

where

$$\begin{aligned}
\lambda_1 &= 4\mu_1^*(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \mu_2^* B_1^* r \text{ and } \lambda_2 = 4\nu_1^*(\|\phi\|_{\mathcal{B}_h}^2 \\
& + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \nu_2^* B_2^* r
\end{aligned}$$

and

$$\lambda_3 = \delta_2 + 4(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \tilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2))(\delta_1 + \sum_{k=1}^m L_k).$$

Then

$$\|(\Psi_1 y)(t) + (\Psi_2 y^*)(t)\|_{\mathbb{H}}^2 \leq 4\tilde{M}_T^2\lambda_3 + 4\tilde{M}_S^2T^{2\alpha}\left[\frac{\lambda_1}{\alpha^2} + \frac{\lambda_2}{T(2\alpha-1)}\right] \leq r$$

Hence, we get $\Psi_1 y + \Psi_2 y^* \in \mathcal{B}_r$.

step 2. The map Ψ_1 is continuous on $\mathcal{B}_r$.

let $\{y^n\}_{n=1}^\infty$ be a sequence in $\mathcal{B}_r$ with $\lim y^n \rightarrow y \in \mathcal{B}_r$. Then for $t \in J$ we have

$$\begin{aligned} E\|(\Psi_1 y^n)(t) - (\Psi_1 y)(t)\|_{\mathbb{H}}^2 &\leq 2\|T_\alpha(t)\|^2 E\|g(y^n + \widehat{\phi}) - g(y + \widehat{\phi})\|_{\mathbb{H}}^2 \\ &+ 2 \sum_{0 < t_k < t} \|T_\alpha(t - t_k)\|^2 E\|I_k(y^n(t_k^-) + \widehat{\phi}(t_k^-)) - I_k(y(t_k^-) + \widehat{\phi}(t_k^-))\|_{\mathbb{H}}^2 \end{aligned}$$

Since the functions g and $I_i, i = 1, 2, \dots, m$ are continuous hence

$$\lim_{n \rightarrow \infty} \|(\Psi_1 y^n)(t) - (\Psi_1 y)(t)\|_{\mathbb{H}}^2 = 0$$

which implies that the mapping Ψ_1 is continuous on $\mathcal{B}_r$.

step 3. Ψ_1 maps bounded sets into bounded sets in $\mathcal{B}_r$.

Let us prove that for $r > 0$ there exists a $\widehat{r} > 0$ such that for each $y \in \mathcal{B}_r$ we have $E\|(\Psi_1 y)(t)\|_{\mathbb{H}}^2 < \widehat{r}$ for $t \in J$. Now we have

$$\begin{aligned} E\|(\Psi_1 y)(t)\|_{\mathbb{H}}^2 &\leq 2\|T_\alpha(t)\|^2 E\|(-g(y + \widehat{\phi}))\|_{\mathbb{H}}^2 \\ &+ 2 \sum_{0 < t_k < t} \|T_\alpha(t - t_k)\|^2 E\|I_k(y(t_k^-) + \widehat{\phi}(t_k^-))\|_{\mathbb{H}}^2 \\ &\leq 2\widetilde{M}_T^2 \lambda_4 = \widehat{r} \end{aligned}$$

$$\begin{aligned} \text{Where } \lambda_4 = & 4\delta_1(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \widetilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \delta_2 \\ & + \sum_{k=1}^m 4L_k(\|\phi\|_{\mathcal{B}_h}^2 + l^2(r + \widetilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) \end{aligned}$$

which proves the desired result.

step 4. The map Ψ_1 is equicontinuous.

Let $u, v \in J$, $0 \leq u < v \leq T$, $y \in \mathcal{B}_r$, we obtain

$$\begin{aligned} E\|(\Psi_1 y)(v) - (\Psi_1 y)(u)\|_{\mathbb{H}}^2 &\leq \left[4\delta_1(\|\phi\|_{\mathcal{B}_h}^2 \right. \\ &+ l^2(r + \widetilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)) + \delta_2 \left. \right] \|T_\alpha(v) - T_\alpha(u)\|_{\mathbb{H}}^2 \\ &+ \sum_{k=1}^m 4L_k[\|\phi\|_{\mathcal{B}_h}^2 \\ &+ l^2(r + \widetilde{M}_T^2 E\|\phi(0)\|_{\mathbb{H}}^2)] \|T_\alpha(v - t_k) - T_\alpha(u - t_k)\|_{\mathbb{H}}^2 \end{aligned}$$

Since T_α is strongly continuous and it allows us to conclude that

$\lim_{u \rightarrow v} \|T_\alpha(v - t_k) - T_\alpha(u - t_k)\|_{\mathbb{H}}^2 = 0$ which implies that $\Psi_1(\mathcal{B}_r)$ is equicontinuous.

Finally, combining Step 1. to Step 4. together with Ascoli's theorem, we conclude that the operator Ψ_1 is compact.

step 5. Ψ_2 is a contraction mapping

Let $y, y^* \in \mathcal{B}_r$ and $t \in J$ we have

$$\begin{aligned}
& \|(\Psi_2 y)(t) - (\Psi_2 y^*)(t)\|_{\mathbb{H}}^2 \leq \\
& 2E \left\| \int_0^t S_\alpha(t-s) \left[f(s, y_s + \widehat{\phi}_s, B_1(y(s) + \widehat{\phi}(s))) \right. \right. \\
& \quad \left. \left. - f(s, y_s^* + \widehat{\phi}_s, B_1(y^*(s) + \widehat{\phi}(s))) \right] ds \right\|_{\mathbb{H}}^2 \\
& + 2E \left\| \int_0^t S_\alpha(t-s) \left[\sigma(s, y_s + \widehat{\phi}_s, B_2(y(s) + \widehat{\phi}(s))) \right. \right. \\
& \quad \left. \left. - \sigma(s, y_s^* + \widehat{\phi}_s, B_2(y^*(s) + \widehat{\phi}(s))) \right] d\omega(t) \right\|_{\mathbb{H}}^2 \\
& \leq 2 \int_0^t \|S_\alpha(t-s)\| ds \int_0^t \|S_\alpha(t-s)\| E \|f(s, y_s \\
& \quad + \widehat{\phi}_s, B_1(y(s) + \widehat{\phi}(s))) \\
& \quad - f(s, y_s^* + \widehat{\phi}_s, B_1(y^*(s) + \widehat{\phi}(s)))\|_{\mathbb{H}}^2 ds \\
& + 2 \int_0^t \|S_\alpha(t-s)\|^2 E \|\sigma(s, y_s + \widehat{\phi}_s, B_2(y(s) + \widehat{\phi}(s))) \\
& \quad - \sigma(s, y_s^* + \widehat{\phi}_s, B_2(y^*(s) + \widehat{\phi}(s)))\|_{\mathbb{H}}^2 ds \\
& \leq 2\tilde{M}_S^2 \int_0^t (t-s)^{\alpha-1} ds \int_0^t (t-s)^{\alpha-1} \left[\mu_1 \|y(s) - y^*(s)\|_{\mathcal{B}_h}^2 \right. \\
& \quad \left. + \mu_2 E \|B_1(y(s) + \widehat{\phi}(s)) - B_1(y^*(s) + \widehat{\phi}(s))\|_{\mathbb{H}}^2 \right] ds \\
& + 2\tilde{M}_S^2 \int_0^t (t-s)^{2(\alpha-1)} \left[\nu_1 \|y_s - y_s^*\|_{\mathcal{B}_h}^2 \right. \\
& \quad \left. + \nu_2 E \|B_2(y(s) + \widehat{\phi}(s)) - B_2(y^*(s) + \widehat{\phi}(s))\|_{\mathbb{H}}^2 \right] ds \\
& \leq 2\tilde{M}_S^2 T^{2\alpha} \left[\frac{1}{\alpha^2} (\mu_1 l + \mu_2 B_l^*) \right. \\
& \quad \left. + \frac{1}{T(2\alpha-1)} (\nu_1 l + \nu_2 B_2^*) \right] \|y - y^*\|_{\mathcal{B}_b^0}^2
\end{aligned}$$

By the condition (17), we obtain that Ψ_2 is a contraction mapping. Hence, by Krasnoselskii's fixed point theorem we can conclude that the problem (1) has at least one solution on $(-\infty, T]$. This completes the proof of the theorem $\square$

Acknowledgments

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper.

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