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ANALYSIS SWARMING BEHAVIOR OF MULTI-AGENTS SYSTEM

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Abstract: The phenomenon of swarm is a natural phenomenon of motion gather from origin to final destination. With this phenomenon task or a job done together by all agents of the swarm. This paper describes the swarm model with the attractor and repellent function between agents of the swarm. In this paper, we study about stationary of swarm center and stability analysis of the model with Lyapunov method. From the analytical results are obtained that the swarm center are stationary. Also, using Lyapunov stability method is obtained that the proposed model is stable. The results of numerical simulations show that all agents will aggregate and enter into in a bounded region around the swarm center. In the last section will show the results of numerical simulation of the swarm behavior.

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1. Introduction

Many natural phenomena are very interesting. Experts try to analyze them

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mathematically. One of the examples is the phenomenon called swarming which occurs in various groups of organism, such as: colonies of bacteria, flocks of birds, schools of fish etc. The flocks of geese create an inverted V formation, when migrating. By doing so, they obtain some advantages, such as the increase of the flying speed up to 24% and the flying distance up to 71% than if each goose flies on its own. Inspired by the flocks of birds, the inverted V flying formation can be used in engineering for cooperative control (multi-robots) and formation control (aircraft and ship). In the literature, some analysis on swarming behaviors have been done. Breder[1] studied the model of animal aggregation with repulsive force and attractive force. Gazi and Passino[3, 4] proposed a swarm model and studied its aggregation, cohesion and stability properties. Chu et all [2] generalized Gazi and Passino model, where they introduced a coupling matrix. The coupling matrix is symmetric, that is the interactions between two agents of the swarm are reciprocal. Shi et all [9] proposed a swarm model analogous with Chu et all model, but the coupling matrix is asymmetric. Miswanto et all in [6] studied the multi-agents system that models swarming behavior. This model is an extension of the model in [2], where perturbation to the system is introduced. Gazi and Passino[5] study the stability of the collective behavior of social foraging swarms, i.e., swarms moving in a profile of nutrient/toxic substances (an attractant/repellent profile). Parrish et all [8] studied some differences in behavior algorithms and aggregation statistics among existing schooling models. Warburton and Lazarus [10] were also considered an individual-based swarm model and studied the effect on cohesion of a family of attraction/repulsion functions. In this paper, swarming behavior of multi-agents is studied. This work is a modification of a model proposed by Miswanto et all in [7]. Analysis of the stationarity of the swarm center and behaviors of swarm agents around the swarm center are reported. In the next section, we present swarm model with multi-agents. In Section 3, we show some numerical simulations to illustrate our results. In Section 4, we expose the future works.

2. The Swarm Model

Consider a swarm model of N agents. This model is a modification of the model in [7], that is the attraction/ repulsion function. The model is as follows

$$\dot{x_i} = \sum_{j=1}^{N} w_{ij} f(x_i - x_j), \quad i = 1, ..., N. \tag{1}$$

Here $\dot{x}_i = \frac{dx_i}{dt}$, and $x_i \in \mathbf{R}^N$ represents the position of the i-th individual and $H = [w_{ij}] \in \mathbf{R}^{NxN}$ is the coupling matrix with w_{ij} elements of nonnegative integer for all $i, j = 1, ..., N$. In this paper, matrix H is assumed to be symmetric, that is $w_{ij} = w_{ji}$ for all i, j and $w_{ii} = 0$ for all $i = 1, .., N$. The symbol $f(.)$ represents the term of attraction and repulsion among members. The attraction/repulsion function is given by

$$f(y) = -y\big(a - b\ln\big(\frac{r}{c + \|y\|^2}\big)\big), \tag{2}$$

where a, b, c and r are positive constant with $b > a$ and $\|y\| = \sqrt{y^T y}$ is the Euclidean norm. The parameter a represents the attraction and the term $b\ln\big(\frac{r}{c+\|y\|^2}\big)$ represents the repulsion. The function is attractive for large distance and repulsive for small distance. Next, the stationarity of the swarm center and behavior of swarm agents around a certain point called swarm center are stated.

The swarm center is given by

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i. \tag{3}$$

The following theorem states the stationarity of the swarm center.

Theorem 1. *The center of the swarm ($\overline{x}$) described in (3) is stationary for all t.*

Proof. This proof follows from the proof of theorem in [4]

$$\text{Since} \quad \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \text{then} \quad \dot{\overline{x}} = \frac{1}{N}\sum_{i=1}^{N} \dot{x}_i.$$

Thus, we obtain

$$\dot{\overline{x}} = \frac{1}{N}\sum_{i=1}^{N}\Big\{\sum_{j=1}^{N}(w_{ij} f(x_i - x_j))\Big\}.$$

From the definition of the function attraction and repulsion $f(y)$, $f(-y) = -f(y)$ for all $y \in \mathbf{R}^n$. Hence

$$\dot{\overline{x}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}(w_{ij} f(x_i - x_j)) = 0.$$

Hence the swarm center $\overline{x}$ is stationary. This completes the proof.

The following theorem shows that all agents will aggregate and approach a bounded region around the swarm center.

Theorem 2. *Let the swarm model be (1) with an attraction and repulsion functions (2). Then, all agents will aggregate and approach a bounded region*

$$\boldsymbol{B}_\rho = \Big\{x : \sum_{i=1}^{N} ||x_i - \overline{x}||^2 \le \rho^2\Big\}$$

$$\textit{where} \;\; \rho = \frac{b\ K\ M\sqrt{2}}{2\ a\ W}, \quad M = \sum_{i,j=1}^{N} w_{ij}, \quad K \in \mathbf{R}$$

and

$$W = \min_{i \neq j,\ i,\ j=1,2,\ldots,N} (w_{ij}).$$

Proof.

Let $M = \sum_{i,j=1}^{N} w_{ij}$, $W = \min_{i \neq j, i,j=1,2,\ldots,N} (w_{ij})$ and $\beta_{ij} = \ln\Big(\frac{r}{c + \|x_i - x_j\|^2}\Big)$

Let $e_i = x_i - \overline{x}$ thus $\dot{e_i} = \dot{x_i} - \dot{\overline{x}}$. Choose

$V = \sum_{i=1}^{N} v_i$ as a Lyapunov function for the swarm, where $v_i = \frac{1}{2} e_i^T e_i$. Then,

$$\dot{V} = \sum_{i=1}^{N} e_i^T \Big\{ -a \sum_{j=1}^{N} w_{ij}(x_i - x_j) + b \sum_{j=1}^{N} w_{ij}(x_i - x_j)\ \beta_{ij} \Big\}$$

$$= -a \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T w_{ij}(x_i - x_j) + b \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T w_{ij}(x_i - x_j)\ \beta_{ij}.$$

Thus

$$\dot{V} \le -a \sum_{i=1}^{N} \sum_{j=1}^{N} W\ e_i^T (x_i - x_j) + b \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} \|e_i^T\|\ \|x_i - x_j\|\ \beta_{ij}.$$

Since function $\beta_{ij}\ \|x_i - x_j\|$ is a bounded function then there is $K \in \mathbf{R}$ such that $\beta_{ij}\ \|x_i - x_j\| < K$. We have

$$\dot{V} \leq -a\sum_{i=1}^{N}\sum_{j=1}^{N}\ W\ e_i^T(x_i - x_j) + b\ K\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\|e_i\|,$$

$$\leq -a\sum_{i=1}^{N}\sum_{j=1}^{N}\ W\ e_i^T(x_i - x_j) + b\ K\sum_{i=1}^{N} M\|e_i\|.$$

From the definition of center of the swarm, we have

$$\sum_{j=1}^{N} x_j = N\overline{x} \quad \text{then} \quad \sum_{j=1}^{N}(x_i - x_j) = N\ (x_i - \overline{x}) = N\ e_i.$$

Therefore,

$$\dot{V} \leq -N\ a\ W\sum_{i=1}^{N} e_i^T\ e_i + b\ K\ M\ \sum_{i=1}^{N}\|e_i\|$$

$$\leq -N\ a\ W\sum_{i=1}^{N}\|e_i\|^2 + b\ K\ M\ \sum_{i=1}^{N}\|e_i\|.$$

Since $\|e_i\|^2 \leq 2V$ then $\|e_i\| \leq \sqrt{2V}$, we have

$$\dot{V} \leq -NaW\ 2V + bKMN\ \sqrt{2V}.$$

$$\dot{V} \leq -N\ V^{1/2}\Big\{2\ a\ W\ V^{1/2} - b\ K\ M\ \sqrt{2}\Big\}.$$

$$\dot{V} <\ 0\ \ \text{then}\ \ V \geq \left\{\frac{b\ K\ M\sqrt{2}}{2\ a\ W}\right\}^2.$$

This completes the proof.

The following theorem give an estimate of the necessary time for the agents to enter a certain ϵ-annulus around the swarm center.

Theorem 3. Let the swarm model be described by the model (1) with the attraction and repulsion function (2). Let $\boldsymbol{B}_\rho$ *be as in theorem 2. Then, for all* $\epsilon > 0$*, all agents out of the set* $\boldsymbol{B}_\rho$ *will aggregate and enter the region* $\epsilon - annulus$ $(\boldsymbol{B}_\epsilon)$*, where*

$$\boldsymbol{B}_\epsilon = \Big\{x : \sum_{i=1}^{N}||x_i - \overline{x}||^2 \leq (\rho + \epsilon)^2\Big\}$$

in a finite time bounded by

$$T = \frac{1}{NaW}\ \ln\left\{\frac{\rho_0 - \rho}{\epsilon}\right\},\ \textit{here}\ \rho_0 = \left\{\sum_{i=1}^{N} \|x_i(0) - \overline{x}(0)\|^2\right\}^{1/2} > \rho + \epsilon.$$

Proof. If an agent i enters the region $\mathbf{B}_\epsilon$ at time T, then

$$\|e_i(T)\| = \rho + \epsilon \ \text{ and } \|e_i(T)\| = \rho + \epsilon \leq \sqrt{2V(T)}.$$

From solution inequality

$$\dot{V} \leq -NaW\ 2V + bKNM\ \sqrt{2V},$$

we have

$$\frac{\epsilon}{\rho_0 - \rho} \leq \left\{\frac{\sqrt{2V(T)} - \rho}{\sqrt{2V(0)} - \rho}\right\} \leq \sqrt{2V(T)} - \rho \leq e^{-NaWT}.$$

Such that

$$ln\left\{\frac{\epsilon}{\rho_0 - \rho}\right\} \leq -NaWT.$$

We have

$$T \leq \frac{1}{NaW}\ ln\left\{\frac{\rho_0 - \rho}{\epsilon}\right\}.$$

This completes the proof.

3. Numerical Simulation

In this section, some numerical simulations to illustrate model (1) are reported. Figure 1-2 show the simulation results, where $N = 10, a = 1, b = 20, c = 1$ and $r = 1.5$. Figure 3-4 show the simulation results, where $N = 10, a = 4, b = 20, c = 1$ and $r = 1.5$. The coupling matrix H is generated randomly by nonnegative integer (0-10) and is symmetric. This program run the simulation for 100 seconds. Figure 1 shows the numerical simulations of the trajectories of the members of the swarm. It can be seen from Fig.1 that the swarm members aggregate towards a bounded region. Then, they continuously move together in a spiral motion. Figure 2 shows the trajectories of the the swarm center. It can be seen, that the swarm center is stationary. Figure 3 shows the numerical simulations the trajectories of the member of the swarm model. It can be seen from Fig.3, that the swarm members will aggregate towards a bounded region.

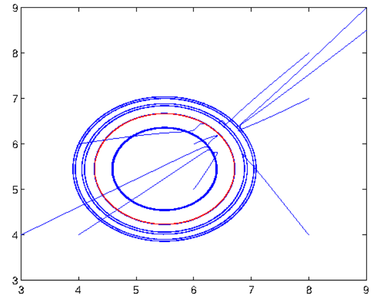

Figure 1: The trajectories of the agent of the swarm model. Number 1, 2, 3, etc denote agent of the swarm model.

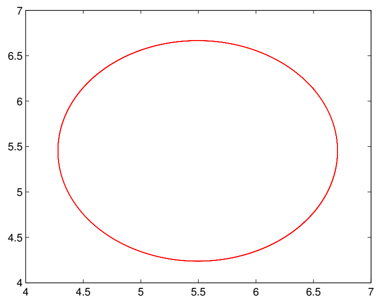

Figure 2: The trajectories of the agent of the swarm model. Number 1, 2, 3, etc denote agent of the swarm model.

Then, they continuously move together in a spiral motion but the size of the swarm is smaller than Fig.1. Figure 4 shows the trajectories of the the swarm center. It can be seen, that the swarm center is stationary but the radius of the swarm center is smaller than Fig.1.

4. Future Works

The analysis method for the swarming behavior of multi-agents under certain condition on the coupling matrix have been presented in this paper. From the numerical simulations results, the swarms center is still stationary and the swarm agents aggregate and enter a certain .-annulus around the swarms center. In the future works, it is interesting to apply this model in engineering field such

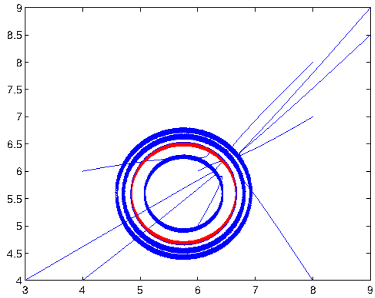

Figure 3: The trajectories of the agent of the swarm model. Number 1, 2, 3, etc denote agent of the swarm model.

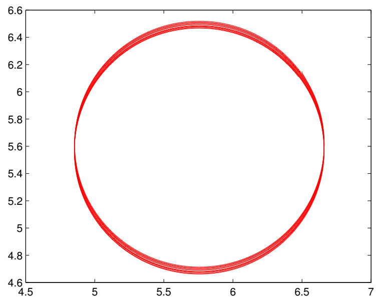

Figure 4: The trajectories of the swarm center

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as in flying inverted "V" formation of the airplane, ships, etc.

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