International Journal of Differential Equations and Applications
Volume 14 No. 4 2015, 369-382
ISSN: 1311-2872
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijdea.v14i4.932

acadpubl.eu

CONFLUENCE OF SHOCKS IN BURGERS EQUATION. A NEW APPROACH

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Abstract: In this paper, an approach based on the infintesimal technics of non standard analysis with a new language for the behaviour formulation based on the evaluations of integrals is proposed to study asymptotic behavior as ε is infinitely small of solutions of boundary value problem for viscid Burgers equation:

$$\begin{cases} u_t + uu_x = \varepsilon u_{xx}, & \forall\, x \in R, \quad t > 0, \\ u(\xi, 0) = f(\xi), & t = 0, \end{cases}$$

where $f : R \to R$, continuous function. We investigate the confluences shocks case. We show that the exact solution of boundary value problem for viscid Burgers equation as ε is sufficiently small, appoach the shock type solution of boundary value problem for inviscid Burgers equation.

$$\begin{cases} u_t + uu_x = 0, & \forall\, x \in R, \quad t > 0, \\ u(\xi, 0) = f(\xi), & t = 0. \end{cases}$$

The results are formulated in classical mathematics and proved with infintesimal technics of non standard analysis.

AMS Subject Classification: 35XX, 35LXX, 58J45
Key Words: non standard analysis, boundary problem, viscid Burgers equation, inviscid Burgers equation, heat equation, confluence of shocks

Received: May 26, 2015

1. Introduction

One of the most important PDEs in the theory of non linear consevation laws is Burgers equation. she combining both nonlinear propagation effects and diffusive effect. This equation has a large variety of application in the modeling of water in unsaturated soil, dynamics of soil water, statistics of flow problems mixing and turbulent diffusion cosmology and seismology. Burgers equation has different types and each of them has special application such as [1,2]. This equation is parabolic when the viscous term is included. If the viscous term is null the remaining equation is hyperbolic. The viscid (viscous) Burgers equation has the form [11]:

$$u_t + uu_x = \varepsilon u_{xx}, \tag{1.1}$$

where ε is a positif parameter small enough.

When ε is null, this equation approch to the Eulers equations in one dimension who govern the flows of parfect - fluids. If ε is not null, we approch to the Naveir Stokes equations in one dimension. If the viscous term is dropped from the Burgers equation discontinuities may appaer in finite time, even if the initial condition is smooth they give rise to the phenomen of shock waves with important application in physics [5]. This properties make Burgers equation a proper model for testing numerical algorithms in flows where severe gradients or shocks are anticipated [1,6]. Discretization methods are well-knoun technics for sollving Burgers equation. Ascher and Mclachlan established many methods as multisymplectic box sheme. For the boundary value problem, Sinai[13] is intersted to the initial condition case: null on R_-,and Brownian on R_+. She, Aurell and Frich[12] with a numerical calculs particularly examine the initial conditions of Brownian fractionnair type.

To study the initial value problem of Burgers equation, classical methods are based on search of solutions of the reduiced problem to deduce existence and asymptotic behavoiur of the solutions as ε tends to 0, the passasge of the limit is very complicated, but in general the limit exist and it's a solution for the reduced problem (when $\varepsilon = 0$).For ε small the solution $u(x,t)$ is approximated by this limit[7]. Other methods are based on a weak formulation of burgers equation seen as a conservation law satisfied on each of the computational domain called cell or finite volume. Stochastic particle method is so used for.

In this paper, we study solutions of inviscid and viscid Burgers equation with an initial value who admits two inflexions points. we introduce the infintesimal technics of non standard analysis (N.S.A) to give a language for the behaviour formulation based on the evaluations of integrals as follows. It is worth noting that our contribution consists of a direct proof based on both infinitesimal technic of non standard analysis and the Van Den Berg method[10].

The aim of this paper is to describe the asymptotic behaviour of the viscid Burgers solution in boundary value problem with a small parameter ε of the form

$$\begin{cases} u_t + uu_x = \varepsilon u_{xx}, & \forall\, x \in R, \quad t > 0 \\ u(\xi, 0) = f(\xi), & t = 0, \end{cases} \tag{1.2}$$

where ε is a positif parameter small enough, f is the initial condition it's a real S continious function. If $\varepsilon = 0$, we obtain the inviscid Burgers value problem(Reduced problem)

$$\begin{cases} u_t + uu_x = 0, & x \in R, \quad t > 0, \\ u(\xi, 0) = f(\xi). & t = 0. \end{cases} \tag{1.3}$$

This problem does not admit the regular solutions but some weak solutions with certain regularity exist.

Our proof is a slight modification to overcome some technical difficulties with considiring this problem. Our general purpose is to show that the exact solution of the boundary value problem (1.2) as ε is infinitely small reduce to solutions endorses the ideas regarding shocks for the reduiced problem (1.3). In this paper, we take up problem (1.2) as a continuity of the above results. Our general purpose is to describe the asymptotic behaviour of solutions in boundary value problem with a small parameter ε and to discuss in particular the case of the confluence shocks through nonstandard analysis.

A simple formulation is given for the asymptotic behavior based on the evaluation of integrals which is a method of the non standard perturbation theory of differential equations proposed by Imm Van Den Berg [10] and improved by Lutz and Callot.

Historically the subject non standard was developed by Robinson, Reeb, Lutz and Gose[7]. The nonstandard perturbation theory of differential equations, which is today a well-established tool in asymptotic theory, has its roots in the seventies, when the Reebian school (see [7,8]) introduced the use of nonstandard analysis into the field of perturbed differential equations. Our goal in this paper is to generalize these techniques on EDP and our general purpose is to describe the asymptotic behaviour of solutions in boundary value problem with a small parameter ε and to discuss in particular the case of confluence shocks with new techniques infinitesimal of non-standard analysis. We can conclude that the solutions of the problem (1.2) is infinitely close to the solutions of (1.3), as ε is a parametr positif sufficiently small.

The paper is organised as follows: In section 2, we start with the Shock Fitting, then we describe the asymptotic behavoir of solution for problem(1.2) in the presence of two shocks (confluence of shocks case). Section 3 contains basic preliminaries results needed in later sections. Section 4 deals with our main result obout confluence shocks case and its proof, we present it in a non standard form.

2. Cauchy Problem for Inviscid Burgers' Equation

2.1. Shock Fitting

Consider the cauchy problem for inviscid Burgers equation:

$$\begin{cases} u_t + uu_x = 0, & \forall\, x \in R, \quad t > 0, \\ u(\xi, 0) = f(\xi). & \qquad\qquad t = 0. \end{cases}$$

Here $f : R \to R.$ is a standarad continuous function.

The Burgers equation on the whole line is known to possess traveling waves solutions. The solution of this problem may be given in a parametric form:

$$\begin{cases} u = f(\xi), \\ x = \xi + f(\xi)t \end{cases} \tag{2.1}$$

and shocks must be fitted in such that:

$$U = \frac{1}{2}(u_1 + u_2) \;= \frac{1}{2}(f(\xi_1) + f(\xi_2), \tag{2.2}$$

where ξ_1 and ξ_2 are the value of ξ on the two sides of the shock, see [4].

According to (2.1), the solution at time t is obtained from the initial profile $u = f(\xi)$ by translating each point a distance $f(\xi)t$ to the right.

The shock cuts out the part corresponding to $\xi_2 \geq \xi \geq \xi_1$. If the discontinuity line, it is a straight line chord betewen the points $\xi = \xi_1$ and $\xi = \xi_2$on the curve $f(\xi)$. Moreover since areas are preserved under the mapping, the equal area property still holds. The cord on the f curve cuts off lobes of equal area. The shock determination can then be describe entirely on the fixe $f(\xi)$ curve by drawing all the chords with the equal area propperty can be written analytically as

$$\frac{1}{2}\left\{(f(\xi_1) + f(\xi_2)\right\}(\xi_1 - \xi_2) = \int_{\xi_2}^{\xi_1} f(\xi)d\xi. \tag{2.3}$$

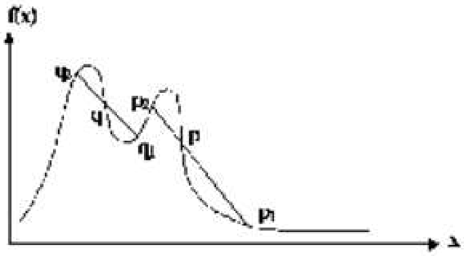

Figure 1: Graphic representation of the initial condition f

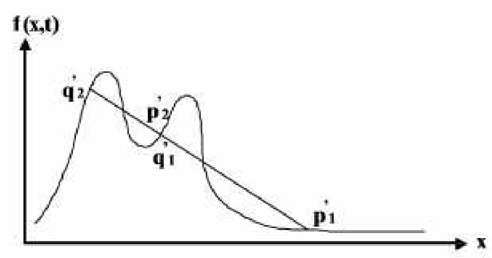

Figure 2: Construction for merging shocks. The caracteristics p_2' and q_1' combined.

This is the differential equation for the shock line cord wich verifies the entropic condition such as in [4].

2.2. Confluence of Shocks

When a number of shocks are produced, it is possible in general for one of them to overtake the shock ahead. They then combine and continue as a single shock. This is also described by our shock solution.

Consider f curve in Figure 1, then two shocks are formed corresponding to the points of inflexions p and q with families of equal area chords, typified by $p_1p_2$and q_1q_2.

As time goes the points $q_1$and p_2 approach each other until the stage in Figure 2 is reached where a common chord cuts off lobes of equal area for both humps.

At this stage the characteristics corresponding to p_2' and q_1' are the same and there fore the shocks have just combined into one as shown in Figure 3.

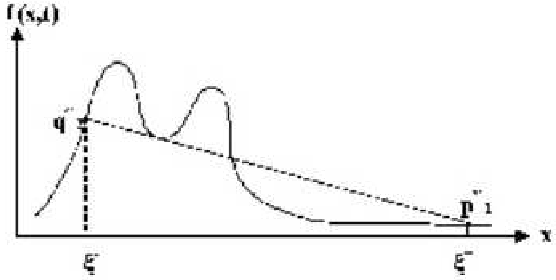

Figure 3: Graphic representation of merging shocks in the final stade. One shock continue.

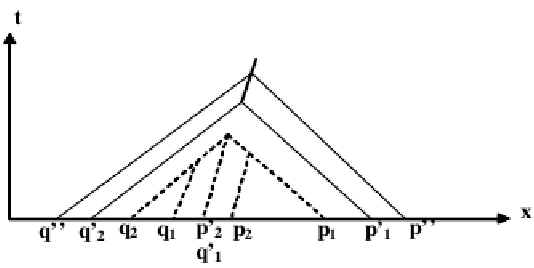

Figure 4: Representation graphique des chocs dans le plan (x, t).

All the characteristics between q_2' and p_1' have now been absorbed by one or other of the shocks. A single shock proceeds using chords $p_1''q_2''$ as in Figure 3 counting only tatal areas above and below the chord in the equal area constuction.

3. Preliminaries Results

In this section we state somme prelimanary results for cauchy problem for viscid burgers' equation, and we give some propositions and lemmas wich will be used in the following.

3.1. Cole-Hopf Transformation

Cole and Hopf noted the remarkable result that (1.1) may be reduced to the linear heat equation ($\varphi_t = \varepsilon\varphi_{xx}$), by the non linear transformation:

$$u = -2\varepsilon[\log \varphi]_x. \tag{3.1}$$

3.2. Confluence of Shocks

When a shock overtakes another shock, they merge into a single shock of increased strength for the inviscid solution ($\varepsilon = 0$).on the f curve in Fig.1. It is possible to give a simple solution of Burgers' equation that descibes this process for arbitrary ε. The solution for a single shock is given in [2,4] and the corresponding expression for the heat equation may be written in the form

$$\varphi = \varphi_1 + \varphi_2, \quad \varphi_i = \exp(-\frac{u_i x}{2\varepsilon} + \frac{u_i^2 t}{4\varepsilon} - b_i). \tag{3.2}$$

In the expression of the solution of one shock, the parametrs b_1, b_2 wich locate the initial position of the shock are taken to be zero. The expressions φ_1, φ_2 are clairement solutions of the heat equation. As consequence we have the following:

Proposition 3.1. *Let $\varphi = \varphi_1 + \varphi_2$, be the analytic solution for the initial value problem for the heat equation, and let f be the initial data. Then u given by the formula* (3.1) *is a solution for the initial value problem* (1.2)., *It is given by:*

$$u = -2\varepsilon\frac{\phi_x}{\phi} = \frac{u_1\varphi_1 + u_2\varphi_2}{\varphi_1 + \varphi_2}. \tag{3.3}$$

Then

$$u(x,t) = \frac{\sum\limits_{I=1}^{i=2} \int_{-\infty}^{+\infty} \frac{x-\eta_i}{t} \exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i} f(\nu)d\nu + \frac{(x-\eta_i)^2}{2t}]]d\eta}{\sum\limits_{i=1}^{i=2} \frac{1}{\sqrt{4\pi\varepsilon t}} \int_{-\infty}^{+\infty} \exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i} f(\nu)d\nu + \frac{(x-\eta_i)^2}{2t}]]d\eta}. \tag{3.4}$$

Proof. When a shock overtakes another shock, they merge into a single shock of increased strength as described in inviscid solution ($\varepsilon = 0$). It is possible to give a simple solution of Burgers equation that describes this process for arbitrary ε.The solution for a single shock is given in [4] and the corresponding expression for φ may be written as in (3.2).

In the solution for a single shock given in[4], The parametrs b_1, b_2 which locate the initial position of the shock are teaken to be zero. The expressions φ_1, φ_2 are clearly solutions of the heat equation ($\varphi_t = \varepsilon\varphi_{xx}$).with

$$\varphi_i = \exp\left(-\frac{u_i}{2\varepsilon}x + \frac{u_i^2}{4\varepsilon}t\right), \quad i = 1, 2, \tag{3.5}$$

corresponding to the initial conditions:

$$\varphi_0 = \phi_0(\eta_i) = \exp\left\{-\frac{1}{2\varepsilon}\int_0^x f(\eta_i)d\eta\right\}, \quad i = 1, 2. \tag{3.6}$$

Then the solutions of the heat equation are given as:

$$\varphi_i = \frac{1}{\sqrt{4\pi\varepsilon t}}\int_{-\infty}^{+\infty}\exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i} f(\nu)d\nu + \frac{(x-\eta_i)^2}{2t}]]d\eta_i, \quad i = 1, 2, \tag{3.7}$$

and

$$u_i\varphi_i = \int_{-\infty}^{+\infty}\frac{x-\eta i}{t}\exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i} f(\nu)d\nu + \frac{(x-\eta_i)^2}{2t}]]d\eta_i, \quad i = 1, 2. \tag{3.8}$$

Using (3.1) and (3.3) we obtain the expression (3.4).

Lemma 3.2. (The Van Den Berg Lemma, see [10]) *Let h be a standard function, definied and inceasing on $[0, +\infty[$ such that $h(v) = av^r\ (1+\delta)$ for $v \simeq 0$, and $h(v) > m(v)^q$. Let φ be an intern function definied on $]0, +\infty[$ such that: $\varphi(v) = bv^s(1+\delta)$ for $v \approx 0$, and such that $\forall\ d > 0$, $\exists$ standard k and c such that: $|\varphi(v)| < k\exp(\cosh(v))$, for $v > d$. Then:*

$$\int_0^\infty \varphi(v)\exp(-\frac{h(v)}{2\varepsilon})dv = \frac{b\Gamma(\frac{(s+1)}{r})}{ra^{\frac{(s+1)}{r}}}\frac{1}{(\frac{1}{2\varepsilon})^{\frac{(s+1)}{r}}}. \tag{3.9}$$

Here, a and *r are positifs standard, m and q are the both positifs. δ is a positif rael small enough. b et s are standard, $b \neq 0$ and $s > -1$.*

To give estimation to the solution, given by (3.3), we state the following lemma:

Lemma 3.3. *Let ε be a positif rael small enough. And let φ and h be two standard functions such that:*

h, is a C^2 class function verifie the lemma 3 , and admits on the ξ point an unique absolute minimum ($h'(\xi) = 0$ et $h''(\xi) > 0$).

$\varphi(\xi) \neq 0$. It is S- *continuous on ξ and satisfie the conditions of the lemma* 3 *in the two sens. Then:*

$$\int_{-\infty}^{+\infty} \varphi(\eta) \exp(-\frac{h(\eta)}{2\varepsilon}) d\eta = \varphi(\xi) \frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi)}}. \exp(-\frac{h(\eta)}{2\varepsilon})(1 + \delta), \tag{3.10}$$

where δ is a positif real small enouhg.

Proof. To prove this lemma, we use the Van Den Berg method, Lemmas (5.6), (5.7) from [10]. It consists in the following stages:

1. To seek the minimum (maximum) absolute of the function under the exponential sign and to make it leave.

2. To bring back the minimum (maximum) to the zero.

3. Search the galaxie where the function wich is under the exponentiel sign is appreciable.

4. Calculate the integral.

We have to state the following lemma.

Lemma 3.4. *Let f the initial condition. Assume:*

(H_1) : $f: R \to R$ is $C^2.(R)$

(H_2) : There exist a, b, c, d and e in R, with $a < b < c < d < e$, such that

$$\begin{cases} f''(x) \geq 0, & \text{if } x \in (]-\infty, a[,\ ]b, c[), \text{ and if } x \in (\ (]c, d],\ ]e, +\infty[)), \\ f''(x) \leq 0, & \text{if } x \in [a, b], \text{ and if } x \in [d, e]. \end{cases}$$

Then for x and t fixed, the functions définied as:

$$h_i(\eta_i, x, t) = \int_0^{\eta_i} f(\nu) d\nu + \frac{(x - \eta_i)^2}{2t},\ i = 1, 2. \tag{3.11}$$

admits every one two minima ξ_i^+ et ξ_i^- for η_i, ($i = 1, 2$) satsfying the equation:

$$x = \eta_i + t f(\eta_i). \tag{3.12}$$

And the condition: $h_i(\xi_i^-, x, t) = h_i(\xi_i^+, x, t)$, *is equivalent to the shock condition:*

$$\frac{1}{2}\left(f(\xi_i^-) + f(\xi_i^+)\right)\left(\xi_i^- - \xi_i^+\right) = \int_{\xi_i^-}^{\xi_1^+} f(\nu)d\nu, \quad i = 1, 2. \tag{3.13}$$

Proof. Let f the initial condition., Two shocks are formed corresponding to the points of inflexions of $f\ (\xi)$. Inside the zone of each shock and for a point (x, t), there are two caracteristcs corresponding to the two minimum who frames a maximum. When a shock overtakes another shock, they merge into a single shock of increased strength as describe for the inviscid solution ($\varepsilon = 0$) on the f curve. The caracteristics are absorbed by one. In this stade there exist two stationnary values which verifies the equations (3.12) we noted them by ξ_i^- and ξ_i^+, $(i = 1, 2)$, each couple frames a maximum. Let $h_i(\eta_i, x, t)$, be the minimum for the functions given by the formula (3.11), is such that:

$$\frac{\partial h_i}{\partial \eta_i} = f(\eta_i) - \frac{x - \eta_i}{t} = 0, \quad i = 1, 2$$

If $h_i(\xi_i^-, x, t) = h_i(\xi_i^+, x, t)$, and within (3.11), this condition can be written as:

$$\int_0^{\xi_i^+} f(\nu)d\nu + \frac{(x - \xi_i^+)^2}{2t} = \int_0^{\xi_i^-} f(\nu)d\nu + \frac{(x - \xi_i^-)^2}{2t}, \quad i = 1, 2 \tag{3.14}$$

But ξ_i^- and ξ_i^+, the both verifies the equations:

$$f(\xi_i) - \frac{x - \xi_i}{t} = 0, \qquad i = 1, 2 \tag{3.15}$$

The shock condition is formulated by the expression(3.14), who it's the same shock condition for the reduced problem.

4. Main Result

In this section we shall present and prove our main result; we discuss the confluences shocks case.

Theorem 4.1. *Under the asymptions* $(H_1),(H_2)$ *in lemma* (3.4), *The problem* (1.2) *has a unique solution for* $t>0$. *given by:*

$$u(x,t)=\frac{\sum_{i=1}^{2}\int_{-\infty}^{+\infty}\frac{x-\eta_i}{t}\exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i}f(\nu)d\nu+\frac{(x-\eta_i)^2}{2t}]]d\eta}{\sum_{i=1}^{2}\frac{1}{\sqrt{4\pi\varepsilon t}}\int_{-\infty}^{+\infty}\exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i}f(\nu)d\nu+\frac{(x-\eta_i)^2}{2t}]]d\eta}. \tag{4.1}$$

Such a solution represents confluence of shocks and for ε *sufficiently small, this solution isinfinitely close to the solution of the reduced problem given in* (2.2).

Proof 1. From figures.2 and 3, after some time the two shocks combine and continue into one, and this is the lowest minimum that carries this amounts to the case of a single shock. From the proposition (3.1), the problem (1.2) admits a single solution explicitemnt given by formula (4.1). Uniqueness is due to the condition of entropy which restricts the set of solutions to one which is stable with a singular perturbation dissipative nature.

2. Let (x, t) be a standard point outside the line of the shock. From the solutions of the equation $x=\eta_i+tf(\eta_i)$, there is only one denoted ξ_i is the absolute minimum of the function given by the expression(3.8). Using the expressions (3.1) and (3.2) of the proposition 3.1 $u(x,t)$ is given as:

$$\begin{aligned}u(x,t)&=\frac{I}{J}\\&=\frac{\sum_{i=1}^{2}u_i\varphi_i}{\sum_{i=1}^{2}\varphi_i}=\frac{\sum_{i=1}^{2}\int_{-\infty}^{+\infty}\frac{x-\eta i}{t}\exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i}f(\nu)d\nu+\frac{(x-\eta_i)^2}{2t}]]d\eta_i}{\sum_{i=1}^{2}\frac{1}{\sqrt{4\pi\varepsilon t}}\int_{-\infty}^{+\infty}\exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i}f(\nu)d\nu+\frac{(x-\eta_i)^2}{2t}]]d\eta_i}.\end{aligned}$$

From the lemma 3.2 we have:

$$u(x,t)=\frac{I}{J}=\frac{\sum_{i=1}^{2}u_i\varphi_i}{\sum_{i=1}^{2}\varphi_i}=\frac{\sum_{i=1}^{2}\frac{x-\eta_i}{t}\frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi_i)}}\exp[-\frac{h(\eta_i)}{2\varepsilon}](1+\delta)}{\sum_{i=1}^{2}\frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi_i)}}\exp[-\frac{h(\eta_i)}{2\varepsilon}](1+\delta)},$$

where $\delta > 0$ is an infinitesimal. So, we have the following estimate

$$u(x,t) = \frac{x-\eta_i}{t}(1+\delta) \simeq u_i(x,t) = f(\xi_i), \quad i = 1,2.$$

To conclude we have the following corollary

Corollary 4.2. *Let (x,t),be a standard point outside each line of shock. Among the solutions of the equation $x = \eta_i + tf(\eta_i)$, one denoted ξ_i is the absolute minimum of the function $h_i(\eta_i, x, t)$ given by*(3.9), *and further the solution of* (1.1) *and* (1.3) *verifies at the point (x,t)*

$$u(x,t) \simeq f(\xi_1) = u_1(x,t), \text{ if } x \text{ is an infinitely large positive}$$
$$u(x,t) \simeq f(\xi_2) = u_2(x,t), \text{ if } x \text{ is an infinitely large negative.}$$

The center of the shock when $\varphi_1 = \varphi_2$ is that:

$$x = (u_1 + u_2)/2t.$$

Proof. Using lemma (3.4), outside the region of each shock. For (x, t) fixed, each function $h_i(\eta_i, x, t)$ has an absolute minimum at $\xi_i (i = 1, 2)$.In (3.1) $u(x,t)$ has the form

$$u(x,t) = \frac{u_1\varphi_1 + u_2\varphi_2}{\varphi_1 + \varphi_2},$$

with φ_i and $u_i\varphi_i$ are given by(3.7) and (3.8). Then

$$u(x,t) = \frac{\displaystyle\sum_{i=1}^{2} \int_{-\infty}^{+\infty} \frac{x-\eta i}{t} \exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i} f(\nu)d\nu + \frac{(x-\eta_i)^2}{2t}]]d\eta_i}{\displaystyle\sum_{i=1}^{2} \frac{1}{\sqrt{4\pi\varepsilon t}} \int_{-\infty}^{+\infty} \exp[\frac{-1}{2\varepsilon}[\int_0^{\eta_i} f(\nu)d\nu + \frac{(x-\eta_i)^2}{2t}]]d\eta_i}.$$

Using lemma (3.3) we obtain

$$u(x,t) = \frac{\left(\frac{x-\eta_1}{t}\frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi_1)}}\exp[-\frac{h(\eta_1)}{2\varepsilon}] + \frac{x-\eta_2}{t}\frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi_2)}}\exp[-\frac{h(\eta_2)}{2\varepsilon}]\right)(1+\delta)}{\left(\frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi_1)}}\exp[-\frac{h(\eta_1)}{2\varepsilon}] + \frac{\sqrt{4\pi\varepsilon}}{\sqrt{h''(\xi_2)}}\exp[-\frac{h(\eta_2)}{2\varepsilon}]\right)(1+\delta)},$$

where δ is an infinitesimal positif real. And it follows that:

If $u_2 > u_1$, φ_1 function dominates when x is infinitely large positive and there is obtained

$$u(x,t) = \frac{x-\eta_1}{t}(1+\delta) \simeq u_1(x,t) = f(\xi_1).$$

If $u_1 > u_2$, φ_2 function dominates when x is infinitely large negative and there is obtained

$$u(x,t) = \frac{x-\eta_2}{t}(1+\delta) \simeq u_2(x,t) = f(\xi_2).$$

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