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A MATHEMATICAL EXPLANATION FOR THE MARTYRDOM EFFECT

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Abstract: Tsebelis and Sprague ([4]), Francisco ([3]), and others adapted predator-prey models from population biology to model the dynamics between a regime and an opposition group (with the regime serving as predator and the opposition group as prey). Meanwhile Lichbach ([2]) and Guitfraind ([1]) considered the internal dynamics of a group as it responded to external coercion. In the following, we join the work of Guitfraind and Lichbach and present a model that suggests that regimes can almost always suppress violent groups, but should move carefully against pacifists.

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1. A Model for Opposition Faction Growth

Let our opposition group be divided into two factions: doves and hawks. The doves use peaceful means (protests, strikes, etc.) to express dissent, while the hawks use violent means (bombings, assassinations). However, both factions are working towards some common goal. We assume the following parameters:

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1. The spontaneous growth rates r, v of the doves and hawks. Note that we need *not* assume these parameters are positive, as society might be so constituted as to reduce the spontaneous accumulation of doves and hawks.

2. The natural growth rates p, u of the doves and hawks. We can view these as the difference between the rate at which doves attract doves (and hawks attract hawks), and the rate at which doves (or hawks) leave the movement. Again, these parameters could be positive or negative.

3. The "contrary growth" rates q, s of the doves and hawks. Roughly speaking, this incorporates factional defection: a dove who becomes disenchanted with the slow pace of peaceful reform, or a hawk who renounces violence. Again, these parameters might be positive or negative.

This leads to the system of differential equations:

$$\frac{dx}{dt} = px + qy + r$$
$$\frac{dy}{dt} = sx + uy + v$$

We might view this as the natural growth pattern of the organization.

In a seminal work, Lichbach ([2]) introduced a crucial change in the dynamical analysis of dissent by treating the mix of strategies used by the opposition as the solution to a constrained optimization problem. Modifying Lichbach's model for our own, let $C(x, y)$ be the societal cost of maintaining an opposition group with x doves and y hawks. This cost need not be paid willingly: whether or not society agrees with the aims of a group of protesters, they impose a societal cost by occupying space that might be used for other purposes. Finally let $B(x, y)$ be the benefit obtained from the regime by the opposition group when it has x doves and y hawks. Our organization seeks to solve (by moving towards the appropriate balance of doves and hawks) one of two optimization problems:

1. Maximize $B(x, y)$, subject to the constraint $C(x, y) = C_0$.

2. Minimize $C(x, y)$, subject to the constraint $B(x, y) = B_0$.

We will designate the optimal solution $(\tilde{x}, \tilde{y})$.

To incorporate this optimal point, let us suppose the leadership is able to guide the growth of the group in some fashion. We will say the leadership is *omniscient* if it knows the location of the optimal point, and *omnipotent* if it has unlimited power to drive the group towards that optimal point. Of course, the leadership of real groups is neither omniscient nor omnipotent, so we suggest a weaker condition: the leadership has an ideal *ratio* of doves to hawks, and some power to drive group membership to that ratio. Note that this ideal ratio need not actually be a solution to the optimization problem; it could be purely ideological (e.g., a leadership dedicated to pacifism would have an ideal ratio of 1 dove to 0 hawks). Let μ, ν positive constants that correspond to the leadership's ability to drive group membership. Then we have our model for the growth of the opposition movement when it is subject to a leadership effect:

$$\frac{dx}{dt} = px + qy + r - \mu(\iota x - \delta y)$$
$$\frac{dy}{dt} = sx + uy + v - \nu(\delta y - \iota x)$$

where Greek letters designate parameters that must be positive. Collecting like terms gives us our final model:

$$\frac{dx}{dt} = ax + by + h$$
$$\frac{dy}{dt} = cx + dy + k$$

Again, we note that there is no *a priori* reason to assign signs to any of these parameters (subject to the constraints given below).

2. Coercion

Lichbach's work implies that a regime has an incentive to act against *any* opposition movement, whether or not it has a table fixed point. This is because an opposition group of any size imposes costs on society and receive benefits from the regime. To model this action against the opposition, we classify regime strategies into two categories: individual and collective coercion. For our purposes, the key difference between the two types is that individual coercion has a more-or-less constant marginal cost, while collective coercion has a decreasing marginal cost.

For example, consider two tactics: arresting the leaders of the opposition, and arresting those who show up at protest rallies. In the first case, arresting the leaders of the opposition requires a certain expenditure of resources; moreover, to arrest the successors requires roughly the same expenditure. Thus this tactic has a (roughly) constant marginal cost, and the cost rises in proportion to the number of members of the opposition affected. Since regime resources are finite, this translates into a fixed number of members of the opposition affected, and would manifest as subtracted positive constants η, κ in our system of differential equations:

$$\frac{dx}{dt} = ax + by + h - \eta$$
$$\frac{dy}{dt} = cx + dy + k - \kappa$$

In the second case, the regime would allocate certain resources (a police presence and a detention area). But once these fixed costs have been paid, the cost of arresting one or a hundred protesters is roughly the same; hence the marginal cost is nominal, and the cost rises only slightly with the number of members of the opposition affected. Thus collective coercion can affect a fixed *fraction* of the opposition. For example, the strategy of arresting protesters would remove doves from the group at a rate proportional to the number of doves in the group (since they would be the ones showing up at protest rallies). This would be reflected by the introduction of positive constants α, β, γ, δ, which would change our system to:

$$\frac{dx}{dt} = (a - \alpha)x + (b - \beta)y + h$$
$$\frac{dy}{dt} = (c - \gamma)x + (d - \delta)y + k$$

Obviously, the regime can pursue both actions. In later papers, we hope to consider the effects of collective coercion, and introduce regime resources as a constraint on the values of α, β, γ, δ, η, and κ.

Note that the individual coercion (the introduction of the constants η and κ) will not change the stability of the fixed point. However, its location may change. From the regime's point of view, there are two concerns. First, the opposition movement might grow as a whole. Second, the opposition movement might radicalize. We will say that coercion has caused a *revitalization* if it leads to growth in both the hawks and doves; *radicalization* if it leads to growth of the hawks and decay of the doves; *moderation* if it leads to growth of the doves and decay of the hawks; and *decay* if it leads to a decay in both the hawk and dove populations.

3. Case Analyses

The fixed point of the system

$$\frac{dx}{dt} = ax + by + h - \eta$$
$$\frac{dy}{dt} = cx + dy + k - \kappa$$

will be located at the intersection of the isoclines, $\bar{x} = \frac{d(\eta-h)-b(\kappa-k)}{ad-bc}$, $\bar{y} = \frac{a(\kappa-k)-c(\eta-h)}{ad-bc}$. Our assumption that the fixed point is stable requires the eigenvalues of the coefficient matrix to have negative real part, imposing the condition $a + d < 0$ and $ad - bc > 0$. Our goal is to assess the effect of individual coercion on the location of the fixed point; hence we note:

$$\frac{\partial \bar{x}}{\partial \eta} = \frac{d}{ad - bc} \qquad \frac{\partial \bar{y}}{\partial \eta} = \frac{-c}{ad - bc}$$

which governs how the fixed point changes when coercion is directed against the doves, and

$$\frac{\partial \bar{x}}{\partial \kappa} = \frac{-b}{ad - bc} \qquad \frac{\partial \bar{y}}{\partial \kappa} = \frac{a}{ad - bc}$$

which governs how the fixed point changes when coercion is directed against the hawks.

Since we are assuming the fixed point is stable, this constrains the possibilities for the signs of a, b, c, and d. There are three cases (each with several sub-cases):

1. Case I: $a, d < 0$. In this case, b and c can have any sign (with certain magnitude constraints), so there are four sub-cases: $b, c < 0$; $b < 0 < c$; $c < 0 < b$; and $b, c > 0$.

2. Case II: $a < 0 < d$. In this case, bc must be negative (since stability requires $ad - bc > 0$). Thus there are two sub-cases: $b < 0 < c$ and $c < 0 < b$.

3. Case III: $d < 0 < a$. As before, bc must be negative, so we again have two sub-cases: $b < 0 < c$ and $c < 0 < b$.

An analysis of these cases leads to Table 1. Note that in most cases, individual coercion against the hawks leads to moderation or decay; in a few cases it

a, d	b, c	vs. Doves	vs. Hawks
$a, d < 0$	$b, c < 0$	R	M
	$b < 0 < c$	D	M
	$c < 0 < b$	R	D
	$b, c > 0$	D	G
$a < 0 < d$	$b < 0 < c$	M	M
	$c < 0 < b$	G	D
$d < 0 < a$	$b < 0 < c$	D	G
	$c < 0 < b$	R	R

Table 1: Effects of Coercion (R = Radicalization, M = Moderation, D = Decay, G = Growth)

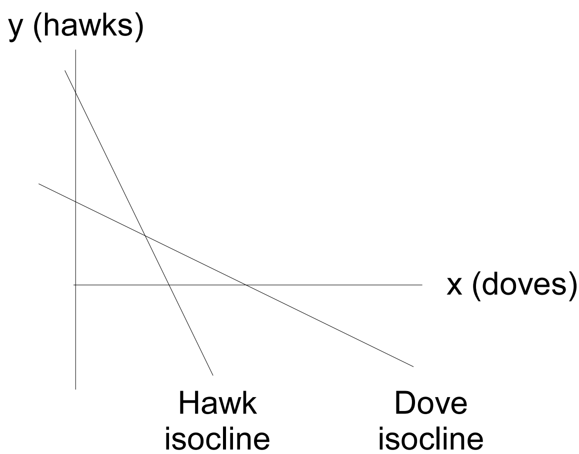

Figure 1: $d < 0 < a$, $c < 0 < b$

leads to growth; and in only one case will it cause radicalization. On the other hand, individual coercion often leads to growth or even radicalization. The message is clear: if the regime *must* act, it is safer to act against the hawks.

What about the one case where acting against the hawks causes radicalization? This will occur when $d < 0 < a$ and $c < 0 < b$. Note that in this case, the requirements of stability and the assumption of signs requires the isoclines to be situated as shown in Figure 1. Since $b > 0$, coercion against doves will raise the dove isocline; since $d < 0$, coercion against hawks will lower the hawk isocline (this is easier to see from the equations giving the isoclines). In both cases, the number of hawks increases while the number of doves decreases; thus both strategies cause radicalization. However, note that raising the dove isocline causes the fixed point to move along the steeper hawk isocline; meanwhile, raising the hawk isocline causes the fixed point to move along the shallower dove isocline. Thus acting against doves has a greater radicalization effect than acting against the hawks. Once again, if the regime must act, it would do better to act against the hawks than against the doves.

The policy implications are clear: if the regime chooses to act against the opposition, it would do better to direct its efforts against the violent faction, and leave the peaceful faction alone. This lends mathematical support to the dictum “He who lives by the sword will die by the sword,” and suggests why the assassination of pacifists rarely benefits a regime.

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