

STABILITY ANALYSIS OF THEILERIOSIS INFECTION WITH ISOLATION

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ABSTRACT: Theileriosis is an infectious disease in cattle caused by theileria annulata, carried by ticks of the genus Hyalomma. A mathematical representation of compartmental model for theileriosis transmission in cattle host and tick vector is formulated. The model incorporates a new class of isolated cattle in the cattle population. Isolated cattle are those that are infected with theileriosis but isolated to prevent further infection. The stability of the model is analysed for the existence of the disease-free and endemic equilibrium points. The basic reproduction number, R_0 , is determined respectively. It is shown that theileriosis infection dies out of cattle population and converges to disease-free equilibrium point if $R_0 < 1$. But if $R_0 > 1$, then the disease persists in the population and converges to endemic equilibrium point.

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1. INTRODUCTION

Ticks of the genus Hyalomma are vector that transmit a protozoan parasite, known

as theileria annulata to cattle. In many parts of the world, *T. annulata* is considered to be a major threat to the cattle industry[9,2]. Theileria sporozoites are transmitted to animals in the saliva of the feeding tick. Ordinarily, *T. annulata* only mature and enter the saliva after the tick attaches to a host. A tick must usually be attached for a few days before it becomes infectious. The incubation period for tropical theileriosis is approximately 1 to 3 weeks and the clinical signs include generalised lymphadenopathy, fever, anorexia and loss of condition with decreased milk yield, jaundice, anemia e.t.c[11].

The biology and epidemiology of theileriosis have been reviewed by a number of authors[5,1,4,13]. However, previous research has concentrated on mathematical modelling of theileriosis infection without considering the importance of isolation in their models. Sutton et al. (2012) worked on mathematical modelling of transmission dynamics of theileria annulata where they described the structure and parametrization of each component of the model in turn, leading to a single infection transmission model. Their model result showed that a peak in the number of clinical cases immediately followed the peak in the number of feeding adult ticks. They also carried out sensitivity analysis to know the impact of the key parameters that describe the transmission of infection between ticks and cattle. They concluded that continuous intervention will help to reduce the transmission of theileriosis in cattle and tick population. Also, Medley et al (1993) analysed the transmission dynamics of theileria parva in Eastern Africa where they gave a method of estimating the rate of infection to cattle of *T.parva* at the endemically stable state. In their analysis, they introduced two control methods influencing the transmission of infection namely: infection and treatment immunization and the reduction in tick feeding by acaricide application.

In view of the above, a deterministic mathematical model to investigate the importance of isolation on transmission dynamics of theileriosis infection in cattle population, which the existing literatures did not consider, is proposed. Isolated cattle are those that are separated voluntarily to prevent further infection.

In addition to the introductory section, the article has four more sections. Section 2 describes the model formulation in details. In Section 3, the existence of solution of the model is obtained. Section 4 deals with the equilibria of the model and basic reproduction number. The detailed analysis and results of the model are in chapter five. Chapter six describes the results and concludes the modelling work.

2. MODEL FORMULATION

In this section, the transmission dynamics of theileriosis will be examined. A theileriosis model incorporating isolated cattle is introduced. The total cattle population N_C is sub-divided into four classes namely; the susceptible cattle, S_C , the infectious cattle, I_C , the isolated cattle, L_C and the recovered cattle, R_C . Also, the total tick population, N_T , is sub-divided into two classes namely; the susceptible tick, S_T and the infected tick I_T . Thus the total population N_C and N_T for cattle and tick population is given by $N_C = S_C + I_C + L_C + R_C$ and $N_T = S_T + I_T$

The population of susceptible cattle is generated by the recruitment of cattle into the population at rate λ_c . It increases when there is transfer of recovery cattle at rate γ due to temporary immunity acquired after successful treatment. It diminishes due to interaction between susceptible cattle and infectious cattle at rates f and g and natural death at rate μ . Putting all these together gives the following equation for the rate of change of the susceptible population:

$$\frac{dS_C}{dt} = \lambda_c N_C - \frac{fgS_C I_T}{N_C} + \gamma R_C - \mu S_C \quad (1)$$

The population of infectious cattle is generated following the fraction of symptomatic cattle that are not isolated at rate ρ . It diminishes due to treatment of infectious cattle at rate α_1 , death due to theileriosis at rate δ and natural death at rate μ . Thus,

$$\frac{dI_C}{dt} = \frac{\rho fg S_C I_T}{N_C} - \alpha_1 I_C - \delta I_C - \mu I_C \quad (2)$$

The population of isolated cattle is generated by the fraction of symptomatic cattle that are isolated at rate $(1 - \rho)$. It diminishes due to treatment at rate α_2 and natural death at rate μ . Thus,

$$\frac{dL_C}{dt} = \frac{(1 - \rho) fg S_C I_T}{N_C} - \alpha_2 L_C - \mu L_C \quad (3)$$

The population of recovered cattle is generated by the treatment of infectious cattle at rate α_1 and treatment of isolated cattle at rate α_2 . It diminishes due to loss of immunity at rate γ and natural death at rate μ . Thus,

$$\frac{dR_C}{dt} = \alpha_1 I_C + \alpha_2 L_C - \gamma R_C - \mu R_C \quad (4)$$

For tick population, we have

$$\frac{dS_T}{dt} = \lambda_t N_T \frac{fq S_T I_C}{N_C} - \mu_T S_T \quad (5)$$

$$\frac{dI_T}{dt} = \frac{fqS_T I_C}{N_C} - \mu_T I_T \quad (6)$$

In (2.5), λ_t denotes the input flow of the susceptible tick including births, f is the infection rate and q is the probability that cattle infect ticks, μ_T is the natural death rate of ticks.

2.1. VARIABLES AND PARAMETERS OF THE MODEL

$S_C(t)$ = the number of susceptible cattle at time t

$I_C(t)$ = the number of infectious cattle at time t

$L_C(t)$ = the number of isolated cattle at time t

$R_C(t)$ = the number of recovered cattle at time t

$S_T(t)$ = the number of susceptible ticks at time t

$I_T(t)$ = the number of infected cattle at time t

f = tick infection rate

g = number of bites that result to infection

q = probability that cattle infect ticks

κ = isolation rate

α_1 = treatment rate of infectious cattle

α_2 = treatment rate of isolated cattle

γ = loss of immunity rate

ρ = fraction of symptomatic that are not isolated

$1 - \rho$ = fraction of symptomatic that are isolated

2.2. ASSUMPTIONS OF THE MODEL

The following assumptions were made in order to formulate the equations of the model:

(a) Due to short duration of theileriosis exposed classes, they do not take part in the dynamics

(b) All newborns are susceptible to infection

(c) Total cattle and tick population are not constant

(d) Recovered cattle have temporary immunity which wanes out and are again susceptible to reinfection

(e) Some infectious cattle that are not isolated remains in the infectious class while some infectious cattle that are isolated progress to isolated class

3. MODEL ANALYSIS

The total population sizes N_C and N_T are

$$\frac{dN_C}{dt} = (\lambda_c - \mu)N_C - \delta i_c, \quad (7)$$

$$\frac{dN_T}{dt} = (\lambda_t - \mu)N_T, \quad (8)$$

which are derived by adding (2.1)-(2.4) for the cattle population and (2.5)-(2.6) for the tick population

3.1. TRANSFORMATION OF THE SYSTEM

It is convenient to use fraction of population instead of population number. This is done by dividing each population class by the total population, and hence, we have:

$$s_c = \frac{S_C}{N_C}, \quad i_c = \frac{I_C}{N_C}, \quad l_c = \frac{L_C}{N_C}, \quad r_c = \frac{R_C}{N_C}, \quad s_t = \frac{S_T}{N_T},$$

$$i_t = \frac{I_T}{N_T}, \quad m = \frac{N_T}{N_C}.$$

Differentiating the fraction with respect to time t gives the following:

$$\frac{ds_c}{dt} = \lambda_c(1 - s_c) - fg\theta s_c i_t + \gamma r_c + \delta s_c i_c, \quad (9)$$

$$\frac{di_c}{dt} = \rho fg\theta s_c i_t - (\alpha_1 + \delta + \lambda_c)i_c + \delta i_c^2, \quad (10)$$

$$\frac{dl_c}{dt} = fg\theta s_c i_t - \rho fg\theta s_c i_t - (\alpha_2 + \lambda_c)l_c + \delta i_c l_c, \quad (11)$$

$$\frac{dr_c}{dt} = \alpha_1 i_c + \alpha_2 l_c - (\gamma + \lambda_c)r_c + \delta r_c i_c, \quad (12)$$

$$\frac{ds_t}{dt} = \lambda_t(1 - s_t) - fq i_c s_t, \quad (13)$$

$$\frac{di_t}{dt} = fq s_t i_c - \lambda_t i_t. \quad (14)$$

Now from $s_c + i_c + l_c + r_c = 1$ and $s_t + i_t = 1$, we have the relation $r_c = 1 - s_c - i_c - l_c$ and $s_t = 1 - i_t$ which give the following system of differential equations:

$$\frac{ds_c}{dt} = \lambda_c(1 - s_c) - fg\theta s_c i_t + \gamma r_c + \delta s_c i_c \quad (15)$$

$$\frac{di_c}{dt} = \rho fg\theta s_c i_t - (\alpha_1 + \delta + \lambda_c)i_c + \delta i_c^2 \quad (16)$$

$$\frac{dl_c}{dt} = fg\theta s_c i_t - \rho fg\theta s_c i_t - (\alpha_2 + \lambda_c)l_c + \delta i_c l_c \quad (17)$$

$$\frac{di_t}{dt} = fqs_t i_c - \lambda_t i_t \quad (18)$$

3.2. EXISTENCE OF SOLUTIONS

Since the model deals with population, all the state variables are assumed to be positive. The invariant region is obtained by the following lemma:

Lemma 1. *The solutions of the system are contained in the region, $\Gamma \in \mathfrak{R}^6$ and $\Gamma_c \cup \Gamma_t \subset \mathfrak{R}_+^4 * \mathfrak{R}_+^2$ (Mtisi et al, 2008).*

Proof. We show that the feasible solutions are uniformly bounded in proper subsets $\Gamma \in \mathfrak{R}_+^6$. Let $(s_c, i_c, l_c, r_c, s_t, i_t) \in \mathfrak{R}^6$ be any solution of the system given by $N_c = s_c + i_c + l_c + r_c$ and $N_t = s_t + i_t$ with non-negative initial conditions. In differential form, we write

$$\frac{dN_c}{dt} = \frac{ds_c}{dt} + \frac{di_c}{dt} + \frac{dl_c}{dt} + \frac{dr_c}{dt}$$

$$\frac{dN_c}{dt} = \lambda_c - \lambda_c N_c + \delta i_c N_c - \delta i_c$$

since

$$s_c + i_c + l_c + r_c = N_c$$

$$\frac{dN_c}{dt} = \lambda_c - (\lambda_c - \delta i_c)N_c - \delta i_c$$

Hence we have

$$\frac{N_c}{dt} + (\lambda_c - \delta i_c)N_c = \lambda_c - \delta i_c$$

Solving the above equation using integrating factor yields

$$N_h = 1 + Be^{-(\lambda_c - \delta i_c)t} \quad (19)$$

Applying the initial condition $N_c(0) = N_c^o$ leads to

$$N_c = 1 + (N_c^o - 1)e^{-(\lambda_c - \delta i_c)t} \quad (20)$$

Thus $N_c \rightarrow 1$ as $t \rightarrow \infty$

And

$$\frac{dN_t}{dt} = \lambda_t - \lambda_t N_t$$

$$\frac{dN_t}{dt} + \lambda_t N_t = \lambda_t$$

We solve to obtain

$$N_t = 1 + (N_t^o - 1)e^{-\lambda_t t}$$

Thus $N_t \rightarrow 1$ as $t \rightarrow \infty$.

Hence the feasible region for the model is given by

$$\Gamma = (s_c, i_c, l_c, r_c, s_t, i_t) \in \mathfrak{R}_+^6; s_c, i_c, l_c, r_c, s_t, i_t \geq 0, s_c + i_c + l_c + r_c = 1; s_t + i_t = 1$$

which is positively invariant set for the model system. Hence, the model is well-posed and biologically realistic and meaningful.

4. EQUILIBRIA OF THE MODEL AND BASIC REPRODUCTION NUMBER

Equilibria points are points where the derivatives are zero. Hence the equilibria points of the system are obtained by setting the right-hand side of (3.9)-(3.12) to zero and the system takes the form

$$\lambda_c(1 - s_c) - fg\theta s_c i_t + \gamma(1 - s_c - i_c - l_c) + \delta s_c i_c = 0, \quad (21)$$

$$\rho fg\theta s_c i_t - (\alpha_1 + \delta + \lambda_c)i_c + \delta i_c^2 = 0, \quad (22)$$

$$fg\theta s_c i_t - \rho fg\theta s_c i_t - (\alpha_2 + \lambda_c)l_c + \delta i_c l_c = 0, \quad (23)$$

$$fq i_c(1 - i_t) - \lambda_t i_t = 0. \quad (24)$$

4.1. BASIC REPRODUCTION NUMBER

The computation of the basic reproduction number R_o is needed in order to assess the stability of disease-free and endemic equilibrium. To know how many infectious individuals are generated by a single infectious one into a susceptible population, the equations of the system are written beginning with the infectious classes and use the next generation matrix to determine the basic reproduction number. We obtain the reproduction number R_o by expressing (3.9)-(3.12) as the difference between the rate of new infection in each infected compartment F and the rate of transfer between each

infected compartment G. Hence, we have

$$\begin{bmatrix} \frac{di_c}{dt} \\ \frac{dq}{dt} \\ \frac{dl_c}{dt} \\ \frac{di_t}{dt} \end{bmatrix} = F - G = \begin{bmatrix} \rho f g \theta s_c i_t \\ f g \theta s_c i_t \\ f q s_t i_c \end{bmatrix} - \begin{bmatrix} (\alpha_1 + \delta + \lambda_c) i_c + \delta i_c^2 \\ \rho f g \theta s_c i_t + (\alpha_2 + \lambda_c) l_c + \delta i_c l_c \\ \lambda_t i_t \end{bmatrix}$$

The Jacobian matrices J_F and J_G of F and G are found about E_0 .

$$S = J_F J_G^{-1} = \begin{bmatrix} 0 & -\frac{f^2 q \rho g \theta}{(\alpha_2 + \lambda_c) \lambda_t} & \frac{f q}{\lambda_t} \\ 0 & \frac{f g \theta}{\alpha_2 + \lambda_c} & 0 \\ \frac{\rho f g \theta}{\alpha_1 + \delta + \lambda_c} & 0 & 0 \end{bmatrix}$$

R_o is the maximum eigenvalue of S given as

$$R_o = \frac{\rho q \theta g f^2}{B_T \lambda_t},$$

where

$$B_T = \alpha_1 + \delta + \lambda_c.$$

5. STABILITY ANALYSIS

5.1. GLOBAL STABILITY OF THE DISEASE-FREE EQUILIBRIUM

Theorem 1. *The disease-free equilibrium E_o of (3.9)-(3.12) is globally asymptotically stable in Γ if $R_o \leq 1$ and unstable if $R_o > 1$.*

Proof. Consider the Lyapunov function $L = f q i_c + B_T i_t$. Its time derivative is

$$\begin{aligned} L' &= f q \frac{di_c}{dt} + B_T \frac{di_t}{dt} \\ &= \rho f^2 g \theta q s_c i_t - f q i_c [B_T - \delta i_c] + B_T [f q i_c (1 - i_t) - \lambda_t i_t] \\ &= \rho f^2 g \theta q s_c i_t - B_T \lambda_t i_t + f q i_c^2 \delta - f q i_c B_T + B_T f q i_c - B_T f q i_c i_t \\ &= B_T \lambda_t i_t \left(\frac{\rho f^2 g \theta q s_c}{B_T \lambda_t} - 1 \right) + f q i_c (\delta i_c - B_T i_t) \\ &= B_T \lambda_t i_t (R_o s_c - 1) - f q i_c (B_T i_t - \delta i_c) \\ &\leq B_T \lambda_t i_t (R_o s_c - 1) \leq 0 \quad \text{if } R_o \leq 1. \end{aligned}$$

Therefore, $L' \leq 0$ for $R_o \leq 1$. One sees further that $(s_c, i_c, l_c, i_t) \rightarrow (1, 0, 0, 0)$ as $t \rightarrow \infty$. Consequently, the largest compact invariant set in $\{(s_c, i_c, l_c, i_t) \in \Gamma : L' = 0\}$ is the E_0 and by Lyapunov-Lasalle's Theorem(see[J.K. Hale, 1969]), the disease-free equilibrium point is globally asymptotically stable in Γ if $R_o \leq 1$ and this completes the proof of Theorem 1.

5.2. EXISTENCE AND UNIQUENESS OF ENDEMIC EQUILIBRIUM E_1

For the existence and uniqueness of endemic equilibrium $E_1 = (s_c^*, i_c^*, l_c^*, i_t^*)$, its coordinates should satisfy the conditions $s_c^* > 0, i_c^* > 0, l_c^* > 0, i_t^* > 0$.

Adding (4.1)-(4.4), we have

$$\begin{aligned} \lambda_c(1 - s_c^* - i_c^* - l_c^*) + \gamma(1 - s_c^* - i_c^* - l_c^*) - \delta i_c^*(1 - s_c^* - i_c^* - l_c^*) \\ - (\alpha_1 i_c + \alpha_2 l_c) + f q i_c(1 - i_t) - \lambda_t i_t = 0. \end{aligned}$$

From Eq. (4.4), $f q i_c(1 - i_t) - \lambda_t i_t = 0$. This gives

$$(\lambda_c + \gamma - \delta i_c^*)(1 - s_c^* - i_c^* - l_c^*) = \alpha_1 i_c + \alpha_2 l_c,$$

since $(1 - s_c^* - i_c^* - l_c^*) > 0$ and from $\delta i_c^* < (\lambda_c + \gamma)$ implies $i_c^* < \frac{\lambda_c + \gamma}{\delta}$.

Thus, an endemic equilibrium point exists, where i_c^* lies in the interval

$$\left(0, \min \left\{1, \frac{\lambda_c + \gamma}{\delta}\right\}\right).$$

We shall use the additive compound matrices approach as in [Mouldowney,(1990); Li et. al.,(1995)] to analyse the stability of endemic equilibrium. We first compute the Jacobian matrix J_E of (4.1)-(4.4).

At the steady state, the Jacobian matrix is given by

$$J_E = \begin{bmatrix} (\lambda_c + \gamma + f g \theta i_t - \delta i_c) & \delta s_c - \gamma & -\gamma & -f g \theta s_c \\ \rho f g \theta i_t & -B_T + 2\delta i_c & 0 & \rho f g \theta s_c \\ f g \theta i_t - \rho f g \theta i_t & \delta l_c & -A_T + \delta i_c & f g \theta s_c - \rho f g \theta s_c \\ 0 & f q(1 - i_t) & 0 & -\lambda_t - f q i_c \end{bmatrix} \quad (25)$$

where

$$A_T = \alpha_2 + \lambda_c.$$

From the Jacobian matrix, the first additive compound matrix is given by

$$J_E^{[1]} = \begin{bmatrix} -(K - \delta i_c^*) & 0 & 0 & 0 \\ 0 & -(M - 2\delta i_c^*) & 0 & 0 \\ 0 & 0 & -(N - \delta i_c^*) & 0 \\ 0 & 0 & 0 & -(X) \end{bmatrix} \quad (26)$$

where

$$K = \lambda_c + \gamma + fg\theta i_t, \quad M = \alpha_1 + \delta + \lambda_c,$$

$$N = \alpha_2 + \lambda_c, \quad X = \lambda_t + fq i_c.$$

5.3. GLOBAL STABILITY OF ENDEMIC EQUILIBRIUM E_1

We state and prove conditions for global stability of endemic equilibrium E_1 in what follows:

Theorem 2. *For $n = 4$ and D convex and bounded and suppose that (3.9)-(3.12) is competitive, permanent and have the property of stability of periodic orbits. If \tilde{x}_0 is the only equilibrium point in $\text{int}D$, and if it is locally asymptotically stable, then it is globally asymptotically stable in $\text{int}D$.*

Proof. According to (Mouldowney, et al.,(1995)), the asymptotic orbital stability of a periodic orbit of a general autonomous system, it is sufficient to prove that the linear non-autonomous system

$$w'(t) = (J_E^{[1]}(p(t)))w(t)$$

is asymptotically stable, where $J_E^{[1]}$ is the first additive compound matrix of the Jacobian matrix J_E .

From the first additive compound matrix in (5.2), we have a linear system with respect to the solutions $(s_c(t), i_c(t), l_c(t), i_t(t))$ written as

$$w_1'(t) = -(K - \delta i_c(t))w_1(t), \quad (27)$$

$$w_2'(t) = -(M - 2\delta i_c(t))w_2(t), \quad (28)$$

$$w_3'(t) = -(N - \delta i_c(t))w_3(t), \quad (29)$$

$$w_4'(t) = -Xw_4(t). \quad (30)$$

In order to prove that the system $J_E^{[1]}$ is asymptotically stable, we shall use the following Lyapunov function that is positive but not differentiable everywhere:

$$V(w_1(t), w_2(t), w_3(t), w_4(t)) = \sup\{|w_1|, \frac{i_c(t)}{l_c(t)}(|w_2| + |w_3|), \frac{i_c(t)}{i_t(t)}(|w_4| + |w_5|)\}$$

Denoting the left- hand side derivative of $V(t)$ by $D_+V(t)$, we get the following inequalities:

$$D_+(|w_1(t)|) \leq -(K - \delta i_c(t))|w_1(t)| \tag{31}$$

$$D_+(|w_2(t)|) \leq -(M - 2\delta i_c(t))|w_2(t)| \tag{32}$$

$$D_+(|w_3(t)|) \leq -(N - \delta i_c(t))|w_3(t)| \tag{33}$$

$$D_+(|w_4(t)|) \leq X|w_4(t)| \tag{34}$$

We also have

$$\begin{aligned} D_+ \frac{i_c(t)}{l_c(t)} (|w_2(t)| + |w_3(t)|) &= \left[\frac{i'_c(t)}{i_c(t)} - \frac{l'_c(t)}{l_c(t)} \right] \frac{i_c(t)}{l_c(t)} (|w_2(t)| + |w_3(t)|) \\ &\quad + \frac{i_c(t)}{l_c(t)} D_+(|w_2(t)| + |w_3(t)|) \end{aligned} \tag{35}$$

Adding (5.4) and(5.5), we have

$$\begin{aligned} D_+(|w_2(t)| + |w_3(t)|) &= -(\alpha_1 + \delta + \lambda_c - 2\delta i_c(t))|w_2(t)| - (\alpha_1 + \delta + \lambda_c - 2\delta i_c(t) - \alpha_1 + \delta i_c + \alpha_2)|w_3(t)| \\ &\leq -(\alpha_1 + \delta + \lambda_c - 2\delta i_c(t))(|w_2(t)| + |w_3(t)|) \end{aligned} \tag{36}$$

Substituting (5.12) into (5.11) yields

$$\begin{aligned} &D_+ \frac{i_c(t)}{l_c(t)} (|w_2(t)| + |w_3(t)|) \\ &\leq \left[\frac{i'_c(t)}{i_c(t)} - \frac{l'_c(t)}{l_c(t)} \right] \frac{i_c(t)}{l_c(t)} (|w_2(t)| + |w_3(t)|) \\ &\quad + \frac{i_c(t)}{l_c(t)} [-(\alpha_1 + \delta + \lambda_c - 2\delta i_c(t))(|w_2(t)| + |w_3(t)|)] \\ &\leq \frac{i_c(t)}{l_c(t)} (|w_2(t)| + |w_3(t)|) \left[\frac{i'_c(t)}{i_c(t)} - \frac{l'_c(t)}{l_c(t)} - \alpha_1 - \delta - \lambda_c + 2\delta i_c(t) \right] \end{aligned} \tag{37}$$

Next,

$$D_+ \frac{l_c(t)}{i_t(t)} (|w_3(t)| + |w_4(t)|) = \left[\frac{l'_c(t)}{l_c(t)} - \frac{i'_t(t)}{i_t(t)} \right] \frac{l_c(t)}{i_t(t)} (|w_3(t)| + |w_4(t)|)$$

$$+ \frac{l_c(t)}{i_t(t)} D_+(|w_3(t)| + |w_4(t)|) \quad (38)$$

Adding (5.5) and(5.6), we have

$$\begin{aligned} D_+(|w_3(t)| + |w_4(t)|) &= -(N - \delta i_c(t))|w_3(t)| - X|w_4(t)| \\ &= -(\alpha_2 + \lambda_c - \delta i_c(t))|w_3(t)| - (\alpha_2 + \lambda_c - \delta i_c(t) + \alpha_2 + \lambda_c + \delta i_c(t) + \lambda_t + fqi_c)|w_4(t)| \\ &\leq -(\alpha_2 + \lambda_c - \delta i_c(t))(|w_3(t)| - (\alpha_2 + \lambda_c - \delta i_c(t))|w_4(t)|) \\ &\leq -(\alpha_2 + \lambda_c - \delta i_c(t))(|w_3(t)| + |w_4(t)|) \end{aligned} \quad (39)$$

Substituting (5.15) into (5.14) yields

$$\begin{aligned} &D_+ \frac{l_c(t)}{i_t(t)} (|w_3(t)| + |w_4(t)|) \\ &\leq \left[\frac{l'_c(t)}{l_c(t)} - \frac{i'_t(t)}{i_t(t)} \right] \frac{l_c(t)}{i_t(t)} (|w_3(t)| + |w_4(t)|) + \frac{l_c(t)}{i_t(t)} [-(\alpha_2 + \lambda_c - \delta i_c(t))(|w_3(t)| + |w_4(t)|)] \\ &\leq \frac{l_c(t)}{i_t(t)} (|w_3(t)| + |w_4(t)|) \left[\frac{l'_c(t)}{l_c(t)} - \frac{i'_t(t)}{i_t(t)} - \alpha_2 - \lambda_c - \delta i_c(t) \right] \end{aligned} \quad (40)$$

From (5.3),(5.13) and (5.16), we have

$$D_+V(t) \leq \sup(g_1(t), g_2(t), g_3(t))V(t),$$

in which

$$g_1(t) = -(\lambda_c + \gamma + fg\theta i_t - \delta i_c(t)) \quad (41)$$

$$g_2(t) = \frac{i'_c(t)}{i_c(t)} - \frac{l'_c(t)}{l_c(t)} - \alpha_1 - \delta - \lambda_c + 2\delta i_c(t) \quad (42)$$

$$g_3(t) = \frac{l'_c(t)}{l_c(t)} - \frac{i'_t(t)}{i_t(t)} - \alpha_2 - \lambda_c - \delta i_c(t) \quad (43)$$

(5.17),(5.18) and (5.19) simplify to

$$g_1(t) = [\delta i_c(t) - (\lambda_c + \gamma + fg\theta i_t)] \quad (44)$$

$$g_2(t) = \frac{i'_c(t)}{i_c(t)} + \left[2\delta i_c(t) - (\lambda_c + \alpha_1 + \delta + \frac{l'_c(t)}{l_c(t)}) \right] \quad (45)$$

$$g_3(t) = \frac{l'_c(t)}{l_c(t)} + \left[-\delta i_c(t) - (\lambda_c + \alpha_2 + \frac{i'_t(t)}{i_t(t)}) \right] \quad (46)$$

so that

$$\sup\{g_1(t), g_2(t), g_3(t)\} \leq \frac{i'_c(t)}{i_c(t)} + \frac{l'_c(t)}{l_c(t)} - \delta \quad (47)$$

From (5.23), we have

$$\lim_{w \rightarrow +\infty} \int_0^w \sup\{g_1(t), g_2(t), g_3(t)\} \leq \lim_{w \rightarrow +\infty} [\ln i_c(t)]_0^w + \lim_{w \rightarrow +\infty} [\ln l_c(t)]_0^w - \delta w = -\delta w < 0 \quad (48)$$

This shows that the periodic solution $(s_c(t), i_c(t), l_c(t), i_t(t))$ is asymptotically stable. This establishes the fact that the endemic equilibrium point of the disease is globally stable.

6. DISCUSSION AND CONCLUSION

In this work, a mathematical model is formulated and analysed to study the transmission and control of Theileriosis infection. A 6-dimensional system of nonlinear ordinary differential equations is modelled mathematically. It is shown that there exist a domain Γ where the model is well-posed and biologically meaningful. The model incorporates a new class of isolated cattle which are the infectious cattle separated voluntarily to prevent further infection. The disease-free equilibrium points of the model are obtained and analysed for stability. The condition for disease spread which is the basic reproduction number, R_0 , is calculated respectively. It is shown that when $R_0 \leq 1$, theileriosis is cleared from the population. Whereas, if $R_0 > 1$, the disease persists in the population. Conclusively, isolation of infectious cattle will reduce the spread of the disease among cattle.

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