International Journal of Differential Equations and Applications
Volume **18**, No. 1 (2019), pages: 87-98
ISSN (Print): 1311-2872; ISSN (Online): 1314-6084;
url: https://www.ijdea.eu
ijpam.eu

A NOTE ON THE EXTENDED SONG–CHANG–PHAM'S SOFTWARE RELIABILITY MODEL. II.

Nikolay Pavlov[1], Georgi Spasov[2],
Martin Stieger[3], Angel Golev[4],

[1,2,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA
[3]Allensbach University
Lohnerhofstrasse 2, D-78467 Konstanz, GERMANY

ABSTRACT: The determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation - the subject of study in the present paper.

For example we study the Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by cumulative function based on the extended Song–Chang–Pham's model [1] (see, also [2]).

We propose a software module within the programming environment *CAS Mathematica* for the analysis of the considered family of functions.

We give real example with dataset using the new extended Song–Chang–Pham's software reliability model.

AMS Subject Classification: 41A46

Key Words: four parameters extended Song–Chang–Pham's cumulative function (4SHPcdf), Heaviside step–function $h_{t_0}(t)$, Hausdorff distance, upper and lower bounds

Received: May 22, 2019 **Revised:** August 27, 2019
Published: September 11, 2019 **doi:** 10.12732/ijdea.v18i1.8
Academic Publications, Ltd. https://acadpubl.eu

1. INTRODUCTION AND PRELIMINARIES

Detailed description of all elements in the area of debugging theory may be found in the following books [4]–[6]. In the books [7]–[8], we pay particular attention to both deterministic approaches and probability models for debugging theories. A Hausdorff metric was chosen to evaluate the test data which are fitted to the sigmoid models proposed in these book. Some of the existing cumulative distributions (Gompertz–Makeham, Yamada-exponential, Yamada–Rayleigh, Yamada–Weibull, transmuted inverse exponential, transmuted Log-Logistic, Kumaraswamy–Dagum and Kumaras–wamy Quasi Lindley) are considered in the light of modern debugging and test theories. Some software reliability models, can be found in [9]–[34]. In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by function based on the extended Song–Chang–Pham's [1] cumulative function. We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis. The models have been tested with real-world data.

Here we will expose from a study by Song, Chang and Pham [1]. A general mean value function $m(t)$ of software reliability models is given by solution of the equation

$$\frac{d\,m(t)}{dt} = \eta h(t)(a(t) - m(t)),$$

where:

- $m(t)$ is the mean value function of faults detected up to time t;
- $a(t)$ is the total number of faults in the Software at time t;
- $h(t)$ represents the fault detection rate function dependent on time t
- (assume η's probability density function is $g(\eta)$)

and $m(t_0) = m_0$ is the marginal condition of

$$m(t) = \int_0^{+\infty} e^{-xH(t)} \left(m_0 + \int_{t_0}^{t} xa(\tau)h(\tau)e^{xH(\tau)}d\tau \right) g(x)dx,$$

$$H(t) = \int_0^t h(u)du.$$

In [49] we study the Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal function based on the Song, Chang and Pham [1] cumulative function

$$M(t) = N\left(1 - \left(\frac{\beta}{\beta + \ln\frac{a+e^{bt}}{a+1}}\right)^{\alpha}\right). \tag{1}$$

where $a, b, \alpha, \beta > 0,\ t > 0$.

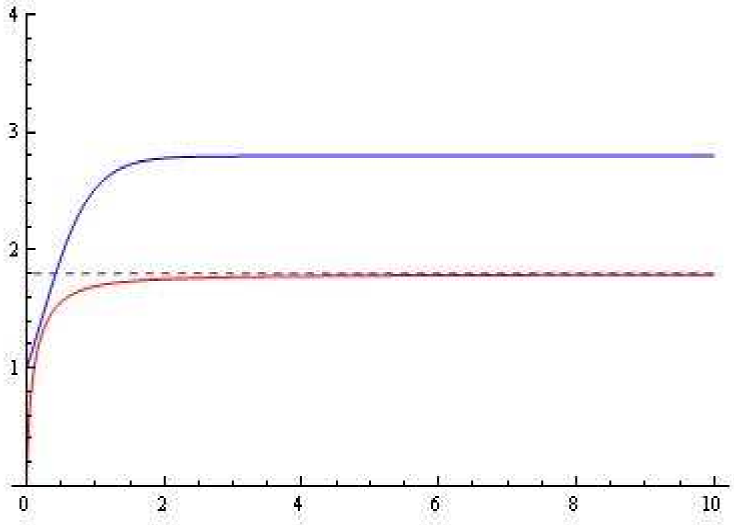

Figure 1: The functions $m(t)$–red; $h(t)$–blue.

Definition 1. *Song, Chang and Pham [1] developed the following new extended software reliability growth model:*

$$M_2(t) = N\left(1 - \frac{\beta}{\beta + \ln \frac{a+e^{bt}}{a+1}}\right)^{\alpha}, \tag{2}$$

where $a, b, \alpha, \beta > 0,\ t > 0$.

Remark. The model (2) is based on the special choice:

$$h(t) = \frac{b}{1 + ae^{-bt}}$$

in the differential equation

$$\frac{d\,m(t)}{dt} = \eta h(t)(a(t) - m(t))$$

(see, Fig. 1).

Typical confidence intervals for the model of Song, Chang and Pham are visualized on Fig. 2.

We study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by function based on the extended Song–Chang–Pham's [1] cumulative function (2).

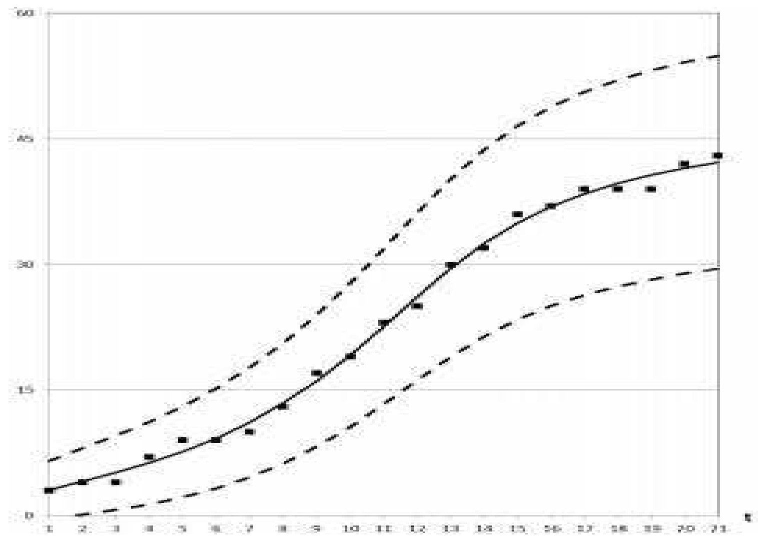

Figure 2: Typical confidence intervals [1].

Definition 2. *The shifted Heaviside step function is defined by*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{3}$$

Definition 3. *[35] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,*

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

2. MAIN RESULTS

2.1. A NOTE ON THE EXTENDED SOFTWARE RELIABILITY GROWTH MODEL (2)

The investigation of the characteristic "supersaturation" of the model (2) to the horizontal asymptote is important.

Without loosing of generality we will look at the following "cumulative sigmoid":

$$M_2^*(t) = \left(1 - \frac{\beta}{\beta + \ln \frac{a+e^{bt}}{a+1}}\right)^{\alpha}, \tag{4}$$

with $N = 1$, and

$$t_0 = \frac{1}{b} \ln \left((1+a) e^{\beta \left(\frac{1}{1-(1/2)^{\frac{1}{\alpha}}} - 1 \right)} - a \right). \tag{5}$$

Evidently, for the "median" we have

$$M_2^*(t_0) = \frac{1}{2}.$$

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the cumulative function (4) satisfies the relation

$$M_2^*(t_0 + d) = 1 - d. \tag{6}$$

For example, for some conditions the following is valid

Let

$$p_1 = -\frac{1}{2},$$

$$q_1 = 1 + \frac{b\alpha e^{bt_0} \left(1 - (1/2)^{\frac{1}{\alpha}}\right)^2}{2\beta(a + e^{bt_0})(1/2)^{\frac{1}{\alpha}}}.$$

For the one–sided Hausdorff distance d between $h_{t_0}(t)$ and the function (4) the following inequalities hold for:

$$2.1 q_1 > e^{1.05}$$

$$d_l = \frac{1}{2.1 q_1} < d < \frac{\ln(2.1 q_1)}{2.1 q_1} = d_r. \tag{7}$$

The proof follows the ideas given in [49] and will be omitted.

The model (4) for $\beta = 0.2$, $\alpha = 0.8$, $a = 16$, $b = 10.1$, $t_0 = 0.128313$ is visualized on Fig. 3.

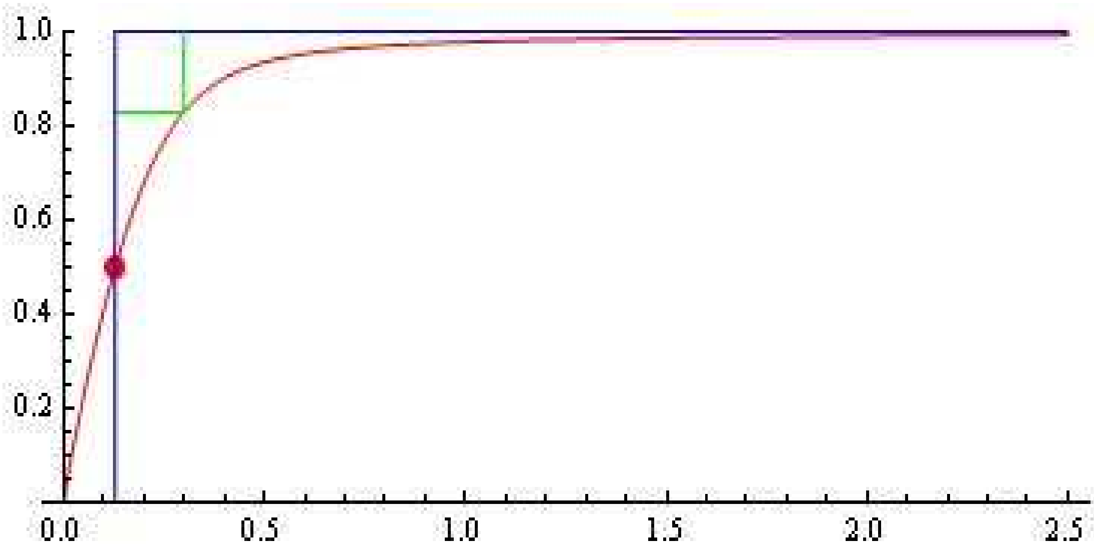

Figure 3: The model (4) for $\beta = 0.2$, $\alpha = 0.8$, $a = 16$, $b = 10.1$, $t_0 = 0.128313$; H–distance $d = 0.170238$, $d_l = 0.119033$ and $d_r = 0.253344$.

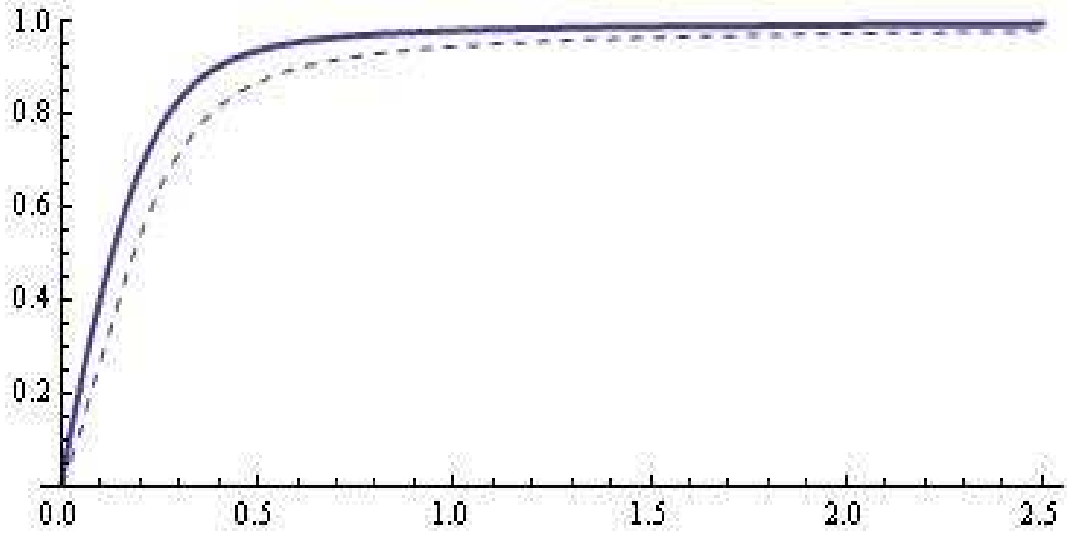

Figure 4: Comparison between the models (1) (dashed) and (4) (blue).

A comparison between the models (1) (for $N = 1$) and (4) for fixed parameters $\beta = 0.2$, $\alpha = 0.8$, $a = 16$, $b = 10.1$ is visualized on Fig. 4.

Test Time (weeks)	CPU hours	Defects Found	PTD G-O	PTD PNZ-SRGM
1	519	16		
2	968	24		
3	1430	27		
4	1893	33		
5	2490	41		
6	3058	49		
7	3625	54		
8	4422	58		
9	5218	69		
10	5823	75	98	74.7
11	6539	81	107	80.1
12	7083	86	116	85.2
13	7487	90	123	90.1
14	7846	93	129	94.6
15	8205	96	129	98.9
16	8564	98	134	102.9
17	8923	99	139	106.8
18	9282	100	138	110.4
19	9641	100	135	111.9
20	10000	100	133	112.2
SSE			12233	495.98
AIC			149.60	138.56

Figure 5: Dataset [26].

2.2. NUMERICAL EXAMPLE

Example. Software Failure Data – Release #1 are presented on Fig. 5 [26].

These data derive from one major release of software products at Tandem Computers [26], [27].

For this data the fitted model for estimated parameters: $N = 100$; $a = 77618.7$; $\beta = 0.000761216$; $\alpha = 0.3873812.9$; $b = 0.4$ is plotted on Fig. 6.

We hope that the results will be useful for specialists in this scientific area. For other results, see [40]–[48].

ACKNOWLEDGMENTS

This paper is supported by the Project FP19-FMI-002 "Innovative ICT for Digital Research Area in Mathematics, Informatics and Pedagogy of Education" of the Scientific Fund of the University of Plovdiv Paisii Hilendarski, Bulgaria.

REFERENCES

[1] K. Song, I. Chang, H. Pham, NHPP software reliability model with inflection factor of the fault detection rate considering the uncertainty of software operating environments and predictive analysis, *Symmetry*, **11** (2019), 521.

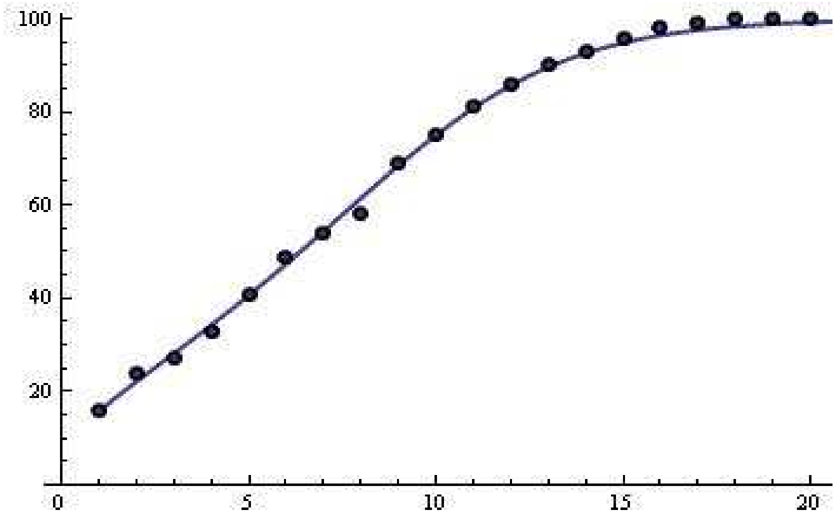

Figure 6: The fitted model $M_2(t)$ with $N = 100$; $a = 77618.7$; $\beta = 0.000761216$; $\alpha = 0.3873812.9$; $b = 0.4$.

[2] Q. Li, H. Pham, A generalized software reliability growth model with consideration of the uncertainty of operating environment, *IEEE Access*, **XX** (2017).

[3] J. D. Musa, A. Ianino, K. Okumoto, Software Reliability: Measurement, Prediction, Applications, McGraw–Hill (1987).

[4] S. Yamada, Software Reliability Modeling: Fundamentals and Applications, Springer (2014).

[5] S. Yamada, Y. Tamura, *OSS Reliability Measurement and Assessment*, In: Springer Series in Reliability Engineering (H. Pham, Ed.), Springer International Publishing Switzerland (2016).

[6] H. Pham, *System Software Reliability*, In: Springer Series in Reliability Engineering, Springer–Verlag London Limited (2006).

[7] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[8] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nontrivial Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-87794-2.

[9] K. Ohishi, H. Okamura, T. Dohi, Gompertz software reliability model: Estimation

algorithm and empirical validation, *J. of Systems and Software*, **82**, No. 3 (2009), 535–543.

[10] D. Satoh, S. Yamada, Discrete equations and software reliability growth models, in: *Proc. 12th Int. Symp. on Software Reliab. and Eng.*, (2001), 176–184.

[11] S. Yamada, A stochastic software reliability growth model with Gompertz curve, *Trans. IPSJ*, **33** (1992), 964–969. (in Japanese)

[12] E. P. Virene, Reliability growth and its upper limit, in: Proc. 1968, *Annual Symp. on Realib.*, (1968), 265–270.

[13] S. Rafi, S. Akthar, Software Reliability Growth Model with Gompertz TEF and Optimal Release Time Determination by Improving the Test Efficiency, *Int. J. of Comput. Applications*, **7**, No. 11 (2010), 34–43.

[14] S. Yamada, M. Ohba, S. Osaki, S–shaped reliability growth modeling for software error detection, *IEEE Trans. Reliab.*, **R–32** (1983), 475–478.

[15] S. Yamada, S. Osaki, Software reliability growth modeling: Models and Applications, *IEEE Transaction on Software Engineering*, **vol. SE–11**, (1985), 1431–1437.

[16] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, Some deterministic reliability growth curves for software error detection: Approximation and modeling aspects, *International Journal of Pure and Applied Mathematics*, **118**, No. 3 (2018), 599–611.

[17] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the Yamada–exponential software reliability model, *International Journal of Pure and Applied Mathematics*, **118**, No. 4 (2018), 871–882.

[18] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note on The "Mean Value" Software Reliability Model, *International Journal of Pure and Applied Mathematics*, **118**, No. 4 (2018), 949–956.

[19] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Analysis of the Chen's and Pham's Software Reliability Models, *Cybernetics and Information Technologies*, **18**, No. 3 (2018), 37–47.

[20] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the generalized inverted exponential software reliability model, International Journal of Advanced Research in Computer and Communication Engineering, 7 (3), 2018, 484–487.

[21] A. L. Goel, Software reliability models: Assumptions, limitations and applicability, *IEEE Trans. Software Eng.*, **SE–11** (1985), 1411–1423.

[22] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Transmuted inverse exponential software reliability model, *Int. J. of Latest Research in Engineering and Technology*, **4**, No. 5 (2018), 1–6.

[23] A. Pandey, N. Goyal, *Early Software Reliability Prediction. A Fuzzy Logic Approach*, In: Studies in Fuzziness and Soft Computing (J. Kacprzyk, Ed.), **303**, Springer, London (2013).

[24] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, A new class of Gompertz–type software reliability models, *International Electronic Journal of Pure and Applied Mathematics*, **12**, No. 1 (2018), 43–57.

[25] O. Rahneva, H. Kiskinov, A. Malinova, G. Spasov, A Note on the Lee-Chang-Pham-Song Software Reliability Model, *Neural, Parallel, and Scientific Computations*, **26**, No. 3 (2018), 297–310.

[26] A. Wood, Predicting software reliability, *IEEE Computer*, **11** (1996), 69–77.

[27] H. Pham, L. Nordmann, X. Zhang, A General Imperfect-Software-Debugging Model with S-Shaped Fault-Detection Rate, *IEEE Trans. Rel.*, **48**, No. 2 (1999), 169–175.

[28] P. K. Kapur, H. Pham, A. Gupta, P. C. Jha, *Software Reliability Assessment with OR Applications*, In: Springer Series in Reliability Engineering, Springer-Verlag, London (2011).

[29] P. Karup, R. Garg, S. Kumar, *Contributions to Hardware and Software Reliability*, World Scientific, London (1999).

[30] M. Lyu (Ed. in Chief), *Handbook of Software Reliability Engineering*, IEEE Computer Society Press, The McGraw-Hill Companies, Los Alamitos (1996).

[31] M. Ohba, Software reliability analysis models, *IBM J. Research and Development*, **21** (1984).

[32] D.R. Jeske, X. Zhang, Some successful approaches to software reliability modeling in industry, *J. Syst. Softw.*, **74** (2005), 85–99.

[33] K. Song, H. Pham, A Software Reliability Model with a Weibull Fault Detection Rate Function Subject to Operating Environments, *Appl. Sci.*, **7** (2017), 983, doi:10.3390/app7100983, 16 pp.

[34] J. D. Musa, *Software Reliability Data*, DACS, RADC, New York (1980).

[35] B. Sendov, *Hausdorff Approximations*, Kluwer, Boston (1990).

[36] N. Kyurkchiev, S. Markov, On the Hausdorff distance between the Heaviside step function and Verhulst logistic function, *J. Math. Chem.*, **54** (2016), 109–119.

[37] L. RogersBennett, D. W. Rogers, S. A. Schultz, Modeling growth and mortality of red abalone Haliotis Rufescens in Northern California, *J. of Shellfish Research*, **26**, No. 3 (2007), 719-727.

[38] C. Zou, *Worms, School Of Electrical Engineering & Computer Science*, Spring (2012).

[39] N. Provos, D. McNamee, P. Mavrommatis, K. Wang, N. Modadugu, The ghost in the browser: Analysis of web-based malware, *In: Proceedings of the 1st Workshop on Hot Topics in Understanding Botnets (HotBots)*, (2007).

[40] N. Kyurkchiev, A. Iliev, A. Rahnev, *Some Families of Sigmoid Functions: Applications to Growth Theory*, LAP LAMBERT Academic Publishing (2019), ISBN: 978-613-9-45608-6.

[41] N. Kyurkchiev, On a Sigmoidal Growth Function Generated by Reaction Networks. Some Extensions and Applications, *Communications in Applied Analysis*, **23**, No. 3 (2019), 383–400.

[42] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the generalized cut functions of degree p+1 by smooth hyper-log-logistic function, *Dynamic Systems and Applications*, **27**, No. 4 (2018), 715–728.

[43] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the Log-logistic and transmuted Log-logistic models. Some applications, *Dynamic Systems and Applications*, **27**, No. 3 (2018), 593–607.

[44] A. Iliev, N. Kyurkchiev, A. Rahnev, T. Terzieva, *Some models in the theory of computer viruses propagation*, LAP LAMBERT Academic Publishing (2019), ISBN: 978-620-0-00826-8.

[45] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, A New Analysis of Cryptolocker Ransomware and Welchia Worm Propagation Behavior. Some Applications. III, *Communications in Applied Analysis*, **23**, No. 2 (2019), 359–382.

[46] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the n-stage growth model. Overview, *Biomath Communications*, **5**, No. 2 (2018), 79–100.

[47] N. Kyurkchiev, S. Markov, *Sigmoid functions: Some Approximation and Modelling Aspects*, LAP LAMBERT Academic Publishing, Saarbrucken (2015), ISBN 978-3-659-76045-7.

[48] N. Kyurkchiev, A. Iliev, S. Markov, *Some techniques for recurrence generating of activation functions*, LAP LAMBERT Academic Publishing (2017), ISBN: 978-3-330-33143-3.

[49] N. Pavlov, G. Spasov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the Song–Chang–Pham's Software Reliability Model. Some Applications. I., *Neural, Parallel, and Scientific Computations*, **27**, No. 2 (2019), 115–129.