International Journal
of Differential Equations and Applications
Volume 11 No. 3 2012, 169-181
ISSN: 1311-8080 (printed version)
url: http://www.ijpam.eu

acadpubl.eu

DEATH OF THE SOLUTIONS OF SYSTEMS DIFFERENTIAL EQUATIONS WITH VARIABLE STRUCTURE AND IMPULSES

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Abstract: Nonlinear non-autonomous systems ordinary differential equations with variable structure and impulses are investigated. The changing of the system right side and impulsive effects take place simultaneously in the so called switching moments. These moments coincide with the moments when the system trajectory cancels the switching functions which are generally nonlinear.

The main objective in the paper is to study the solutions which are not continuable up to infinity. The case of impulsive effects to be the cause of this phenomenon is studied.

AMS Subject Classification: 34A37
Key Words: variable structure, impulses, variable impulsive moments, switching functions

1. Introduction

Applications of differential equations with variable structure (without impulsive effects) are primarily in the control theory: [6], [10], [13], [16], [17], [20], [25] and [28].

Impulsive equations (with fixed structure) are used most commonly for

Received: April 28, 2012

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describing and studying the development of dynamic processes, subjected to the discrete over time external influences: [1], [2], [3], [4], [5], [11], [12], [13], [14], [15], [18], [19], [21], [23], [24], [26], [27] and [29].

Differential equations with variable structure and impulses are introduced in [22]. Some qualitative characteristics of their solutions are studied in [7] and [8]. The equations with variable structure and impulses are used for investigation the dynamics of shutter hydraulic valve in the articles [9] and [13].

The main object of research in this paper is the following initial problem for nonlinear non-autonomous systems of ordinary differential equations with variable structure and impulses in non-fixed moments and nonlinear switching functions:

$$\frac{dx}{dt} = f_i\left(t, x\right), \quad \phi_i\left(x\left(t\right)\right) \neq 0, \ \ t_{i-1} < t < t_i, \tag{1}$$

$$\phi_i\left(x\left(t_i\right)\right) = 0, \quad i = 1,\, 2, ..., \tag{2}$$

$$x\left(t_i + 0\right) \ = \ x\left(t_i\right) \ + \ I_i\left(x\left(t_i\right)\right), \tag{3}$$

$$x\left(t_0\right) = x_0, \tag{4}$$

where:

- The functions $f_i : R^+ \times D \to R^n$, $f_i = \left(f_i^1, f_i^2, ...f_i^n\right)$;
- The phase space D is nonempty domain in R^n;
- The functions $\phi_i : D \to R$;
- The functions $I_i : D \to R^n$;
- $(Id + I_i) : D \to D$;
- The identity in $R^n$is denoted by Id, i.e. $Id\left(x\right) = x$ for $x \in R^n$;
- The initial point $(t_0, x_0) \in R^+ \times D$ and $\phi_1\left(x_0\right) \neq 0$.

The solution of the initial problem considered satisfies:

1.1. For $t_0 \leq t \leq t_1$, the solution of problem (1), (2), (3), (4) coincides with the solution of base problem (1), (4) (with fixed structure and without impulses), i.e. coincides with the solution of problem

$$\frac{dx}{dt} = f_1\left(t, x\right), \ \ x\left(t_0\right) = x_0; \tag{5}$$

1.2. For any t, $t_0 < t < t_1$, it is fulfilled $\phi_1\left(x_1\left(t\right)\right) \neq 0$, where $x_1\left(t\right)$ is the solution of initial problem (5);

1.3. Let t_1 be the first moment after t_0, for which the equality $\phi_1\left(x_1\left(t_1\right)\right) = 0$ is satisfied, i.e. the solution $x_1\left(t\right)$ cancels switching function ϕ_1 at moment t_1;

1.4. At the moment t_1, right side of the studied problem is changing and an impulsive effect on the solution takes place, i.e. the equality (3) for $i = 1$ is satisfied. There is

$$x(t_1 + 0) = x_1(t_1) + I_1(x_1(t_1)) = (Id + I_1)(x_1(t_1));$$

2.1. For $t_1 < t \leq t_2$, the solution of problem (1), (2), (3), (4) coincides with the solution of initial problem

$$\frac{dx}{dt} = f_2(t, x), \quad x(t_1 + 0) = (Id + I_1)(x_1(t_1)); \tag{6}$$

2.2. For any t, $t_1 \leq t \leq t_2$, the inequality $\phi_2(x_2(t)) \neq 0$ is fulfilled, where $x_2(t)$ is a solution of initial problem (6);

2.3. Let t_2 be the first moment after t_1, for which equality $\phi_2(x_2(t_2)) = 0$ is fulfilled;

2.4. At the moment t_2, the structure of the system changes and an impulsive effect takes place, i.e. equality (3) is fulfilled for $i = 2$. There is

$$x(t_2 + 0) = x_2(t_2) + I_2(x_2(t_2)) = (Id + I_2)(x_2(t_2))$$

etc.

The problem solution is continuously on the left side in the moments $t_1, t_2, \ldots$. It means that the solution is left continuously at each point of the maximum interval of existence. On the other hand, in the general case (for example, where the functions $I_i(x) \neq 0$ for $x \in D$, $i = 1, 2, \ldots$) this solution is discontinuously to the right at the points, mentioned above. The jump of the solution at these points is bounded, i.e. there is discontinuity first type. Let us clarify the terms used the points $t_1, t_2, \ldots$ are switching moments, the functions I_i, $i = 1, 2, \ldots$, are named impulsive, and ϕ_i, $i = 1, 2, \ldots$, are switching functions.

Furthermore, we will use the notations:

- $x(t; t_0, x_0)$ is a solution of problem (1), (2), (3), (4);
- $f = \{f_1, f_2, \ldots\}$, $\phi = \{\phi_1, \phi_2, \ldots\}$, $I = \{I_1, I_2, \ldots\}$;
- The sets $\Phi_i = \{x \in D;\ \phi_i(x) = 0\}$, $i = 1, 2, \ldots$, are named switching hypersurfaces of the initial problem considered;
- The function $I_0(x) = 0$ for $x \in D$. The equality $(Id + I_0)(x) = x$ for $x \in D$ is valid;
- $J(t_0, x_0, f_i)$ is the maximum interval of existence of the solution of problem

$$\frac{dx}{dt} = f_i(t, x), \quad x(t_0) = x_0, \quad i = 1, 2, \ldots; \tag{7}$$

• $J(t_0, x_0, f)$ is the maximum interval of existence of the solution of problem (1), (2), (3), (4);

• The function $x_i(t; t_0, x_0)$ is the solution of problem with fixed structure and without impulses (7) for $t \in J(t_0, x_0, f_i)$, $i = 1, 2, ...$;

• The curve $\gamma(t_0, x_0) = \{x(t\,; t_0, x_0)\,,\ \ t \in J(t_0, x_0, f)\}$ is the trajectory of problem investigated;

• The curve $\gamma_i(t_0, x_0) = \{x_i(t\,; t_0, x_0)\,,\ t \in J(t_0, x_0, f_i)\}$ is the trajectory of problem (7);

• $\|\,.\,\|$ and $\langle .,. \rangle$ are Euclidean norm and scalar product in R^n, respectively;

• The set $B_\delta(x_0) = \{x \in R^n;\ \|x - x_0\| < \delta\}$ is δ- neighborhood of point x_0;

• The distance between nonempty sets $A,\ B \subset R^n$ is defined by the equality

$$\rho(A, B) = \inf\{\|x_A - x_B\|\,;\ \ x_A \in A,\ x_B \in B\}\,;$$

• In particular, the distance between point $x_0 \in R^n$ and set $A \subset R^n$ satisfies the equality

$$\rho(x_0, A) = \inf\{\|x_0 - x_A\|\,;\ \ x_A \in A\}\,.$$

Remark 1. Using the solutions of problems with fixed structure and without impulses, we can introduce the solution of problem (1), (2), (3), (4) in the following way:

$$x(t\,; t_0, x_0) = \begin{cases} x_1(t\,; t_0, x_0)\,, \ \ t_0 \le t \le t_1; \\ x_2(t\,; t_1, x(t_1\,; t_0, x_0) + I_1(x(t_1\,; t_0, x_0)))\,, \\ \qquad = x_2(t\,; t_1, x(t_1 + 0\,; t_0, x_0))\,, \ \ t_1 < t \le t_2; \\ \vdots \\ x_{i+1}(t\,; t_i, x(t_i\,; t_0, x_0) + I_i(x(t_i\,; t_0, x_0)))\,, \\ \qquad = x_{i+1}(t\,; t_i, x(t_i + 0\,; t_0, x_0))\,, \ \ t_i < t \le t_{i+1}; \\ \vdots \end{cases}$$

Definition 1. We say that the solution of problem (1), (2), (3), (4) is single right of the initial point t_0, if for any point $t \in J(t_0, x_0, f)$, we have that $x(t\,; t_0, x_0)$ is uniquely determined function.

According to the definition above, it is possible to exist an initial moment $t_0 \in R^+$and points $x_0^*,\ x_0^{**} \in D,\ \ x_0^* \neq x_0^{**}$, such that:

• $x(t\,; t_0, x_0^*)$ is single right of the initial point t_0;

• $x(t\,; t_0, x_0^{**})$is single right of the initial point t_0;

• $(\exists T > t_0):\ (\forall t \in [T, \infty) \cap J(t_0, x_0, f)) \Rightarrow x(t; t_0, x_0^*) = x(t; t_0, x_0^{**})$.

In other words, two or more distinct single right solutions, which start by the same initial moment t_0,but from different initial phase points x_0^* and x_0^{**}, could have a "common part". The single right solutions is permissible to merge. The moments in which, the mergers are made might be points of continuity or discontinuity for each one of the solutions.

Definition 2. We say that the solutions of system (1), (2), (3) die due to the impulsive effects if:

1. It is valid

$$(\forall t_0 \geq 0)\,(\forall x_0 \in D)\,(\forall i = 1, 2, ...) \Rightarrow J(t_0, x_0, f_i) = [t_0, \infty);$$

2. We have

$$(\forall t_0 \geq 0)\,(\forall x_0 \in D)$$

$$(\exists I_1 = I_1(t_0, x_0),\ I_2 = I_2(t_0, x_0), ...;\quad I_1, I_2, ...\ : D \to R^n)$$

$$\left(\exists t^0 = t^0(t_0, x_0, I_1, I_2, ...) = \text{const.} \in R,\ t^0 > t_0\right):$$

$$J(t_0, x_0, f) = \left[t_0, t^0\right).$$

In other words, it is satisfied:

1. For every choice of an initial point in the extended phase space of the system considered and for any fixed right side, which belongs to the set of right sides of basic system (1), the solution of corresponding initial problem with fixed structure and without impulses is continuable up to infinity;

2. For every choice of an initial point in the extended phase space, there exist impulsive functions, such that the solution of the corresponding problem with variable structure and impulses (1), (2), (3), (4) has a limited maximum interval of existence.

The main goal of this consideration is to find the sufficient conditions for death of the system solutions due to the impulsive effects.

The following conditions are introduced:

H1. The functions $f_i \in C[R^+ \times D, R^n]$, $i = 1, 2, ...$.

H2. The functions $\phi_i \in C^1[D, R]$, $i = 1, 2, ...$.

H3. The functions $I_i \in C[\Phi_i, R^n]$ and $(Id + I_i) : \Phi_i \to D$, $i = 1, 2, ...$.

H4. The next inequalities are valid:

$$\phi_i\left(\left(Id+I_{i-1}\right)(x)\right).\left\langle grad\phi_i\left(x\right),f_i\left(t,x\right)\right\rangle<0,\quad (t,x)\in R^+\times D,\ i=1,2,...,$$

where $I_0(x)=0$ for $x\in D$.

H5. There exist the constants $C_{\langle grad\phi_i,f_i\rangle}>0$ such that

$$\big(\forall\,(t,x)\in R^+\times D\big)\Rightarrow\left|\left\langle grad\phi_i\left(x\right),f_i\left(t,x\right)\right\rangle\right|\geq C_{\langle grad\phi_i,f_i\rangle},\quad i=1,2,....$$

H6. For any point $(t_0,x_0)\in R^+\times D$ and for each $i=1,2,...$, the solution of initial problem (7) exists and is unique for $t\geq t_0$.

H7. There exist the constants $C_{\phi_i}>0$ such that

$$\left(\forall x\in\Phi_i\right)\Rightarrow\left|\phi_{i+1}\left(\left(Id+I_i\right)(x)\right)\right|\leq C_{\phi_{i+1}},\quad i=1,2,\ldots.$$

H8. Series $\sum_{i=1}^{\infty}\frac{C_{\phi_{i+1}}}{C_{\langle grad\phi_{i+1},f_{i+1}\rangle}}$ is convergent.

2. Main Results

The following theorems are valid:

Theorem 1. *Let the conditions H1 and H6 be fulfilled.*
Then the solution of problem (1), (2), (3), (4) exists and is unique right from the initial moment t_0.

Theorem 2. *Let the conditions H1–H6 be fulfilled.*
Then the trajectory of problem (1), (2), (3), (4) meets each one of the hypersurfaces $\Phi_i,\ i=1,2,....$

Proof. Firstly, we will show that the trajectory $\gamma_1(t_0,x_0)$ of the initial problem with fixed structure and without impulses (5) meets the hypersurface Φ_1. Recall that

$$\left(Id+I_0\right)(x)=x,\ \ x\in D.$$

Then, using condition H4 (for $i=1$), it follows that one of the following two cases is satisfied:

Case 1. $\phi_1(x)<0$, $x\in D$ and $\left\langle grad\phi_1(x),f_1(t,x)\right\rangle>0$,$(t,x)\in R^+\times D$;
Case 2. $\phi_1(x)>0$, $x\in D$ and $\left\langle grad\phi_1(x),f_1(t,x)\right\rangle<0$, $(t,x)\in R^+\times D$.

Case 1 will be demonstrated. The other case is considered by analogy. For the convenience of recording, we introduce the function

$$\varphi_1(t)=\phi_1\left(x_1\left(t\,;t_0,x_0\right)\right)$$

$$=\phi_1\left(x_1^1\left(t\,;t_0,x_0\right),x_1^2\left(t\,;t_0,x_0\right),...,x_1^n\left(t\,;t_0,x_0\right)\right),$$

defined for $t\in J\left(t_0,x_0,f_1\right)=[t_0,\infty)$. We have

$$\varphi_1\left(t_0\right)=\phi_1\left(x_1\left(t_0;t_0,x_0\right)\right)=\phi_1\left(x_0\right)<0.$$

According to condition H5, it is satisfied

$$\begin{aligned}\frac{d}{dt}\varphi_1\left(t\right)=&\frac{\partial}{\partial x^1}\phi_1\left(x_1\left(t;t_0,x_0\right)\right)\frac{d}{dt}x_1^1\left(t;t_0,x_0\right)\\&+\frac{\partial}{\partial x^2}\phi_1\left(x_1\left(t;t_0,x_0\right)\right)\frac{d}{dt}x_1^2\left(t;t_0,x_0\right)+\cdots\\&+\frac{\partial}{\partial x^n}\phi_1\left(x_1\left(t;t_0,x_0\right)\right)\frac{d}{dt}x_1^n\left(t;t_0,x_0\right)\\=&\frac{\partial}{\partial x^1}\phi_1\left(x_1\left(t;t_0,x_0\right)\right)f_1^1\left(t,x_1\left(t;t_0,x_0\right)\right)\\&+\frac{\partial}{\partial x^2}\phi_1\left(x_1\left(t;t_0,x_0\right)\right)f_1^2\left(t,x_1\left(t;t_0,x_0\right)\right)+\cdots\\&+\frac{\partial}{\partial x^n}\phi_1\left(x_1\left(t;t_0,x_0\right)\right)f_1^n\left(t,x_1\left(t;t_0,x_0\right)\right)\\=&\left\langle grad\phi_1\left(x_1\left(t\,;t_0,x_0\right)\right)\ ,\ f_1\left(t,x_1\left(t\,;t_0,x_0\right)\right)\right\rangle\\=&\left|\left\langle grad\phi_1\left(x_1\left(t\,;t_0,x_0\right)\right)\ ,\ f_1\left(t,x_1\left(t;t_0,x_0\right)\right)\right\rangle\right|\\\geq&C_{\langle grad\phi_1,f_1\rangle}=const>0.\end{aligned}$$

By the fact

$$\varphi_1\left(t_0\right)<0\text{ and }\frac{d}{dt}\varphi_1\left(t\right)=const>0,\quad t>t_0,$$

it follows that, there exists a point $t_1>t_0$ such that

$$\phi_1\left(x_1\left(t_1;t_0,x_0\right)\right)=\varphi_1\left(t_1\right)=0.$$

The meaning is that at the moment t_1, the trajectory $\gamma_1\left(t_0,x_0\right)$ meets hypersurface Φ_1. As

$$\gamma\left(t_0,x_0\right)\equiv\gamma_1\left(t_0,x_0\right)\quad for\ \ t_0\leq t\leq t_1,$$

then, we conclude that the trajectory of the problem with variable structure and impulses also meets the hypersurface Φ_1 at the moment t_1.

Suppose that the trajectory of problem (1) (2) (3) (4) consistently meets the hypersurfaces $\Phi_1,\ \Phi_2,...,\Phi_i$, respectively at the moments $t_1,t_2,...,t_i$. Show that the trajectory

$$\gamma_{i+1}\left(t_i,x\left(t_i;t_0,x_0\right)+I_i\left(x\left(t_i;t_0,x_0\right)\right)\right)\equiv\gamma_{i+1}\left(t_i,x\left(t_i+0;t_0,x_0\right)\right)$$

of the problem with fixed structure and without impulses

$$\frac{dx}{dt} = f_{i+1}(t,x), \quad x(t_i+0) = x(t_i;t_0,x_0) + I_i(x(t_i;t_0,x_0)) = x(t_i+0;t_0,x_0)$$

for $t \in J(t_i, x(t_i+0;t_0,x_0), f_{i+1}) = [t_i,\infty)$ meets the hypersurface Φ_{i+1}. According to condition H4, the functions

$$\phi_{i+1}((Id+I_i)(x)) \text{ and } \langle grad\phi_{i+1}(x), f_{i+1}(t,x)\rangle$$

do not become zero in their domains and for each point $(t,x) \in R^+ \times D$ they possess opposite signs. Without loss of generality, assume that the following inequalities are valid:

$$\phi_{i+1}((Id+I_i)(x)) < 0, \ \ x \in D \text{ and}$$
$$\langle grad\phi_{i+1}(x), f_{i+1}(t,x)\rangle > 0, \ (t,x) \in R^+ \times D.$$

Consider the function $\varphi_{i+1} : [t_i,\infty) \to R$, defined by the equality

$$\begin{aligned}\varphi_{i+1}(t) =& \phi_{i+1}(x_{i+1}(t\,;t_i, x(t_i\,;t_0,x_0) + I_i(x(t_i\,;t_0,x_0))))\\ =& \phi_{i+1}(x_{i+1}(t\,;t_i, x(t_i+0\,;t_0,x_0))). \end{aligned} \tag{8}$$

We have

$$\begin{aligned}\varphi_{i+1}(t_i) =& \phi_{i+1}(x_{i+1}(t_i\,;t_i, x(t_i+0\,;t_0,x_0)))\\ =& \phi_{i+1}(x(t_i+0;t_0,x_0))\\ =& \phi_{i+1}(x(t_i;t_0,x_0) + I_i(x(t_i;t_0,x_0)))\\ =& \phi_{i+1}((Id+I_i)(x(t_i;t_0,x_0))) < 0. \end{aligned} \tag{9}$$

For $t > t_i$, it is fulfilled

$$\begin{aligned}\frac{d}{dt}\varphi_{i+1}(t) =& \frac{d}{dt}\phi_{i+1}(x_{i+1}(t\,;t_i, x(t_i+0\,;t_0,x_0)))\\ =& \langle grad\phi_{i+1}(x_{i+1}(t\,;t_i, x(t_i+0\,;t_0,x_0))), \qquad (10)\\ & f_{i+1}(t, x_{i+1}(t\,;t_i, x(t_i+0\,;t_0,x_0)))\rangle\\ =& |\langle grad\phi_{i+1}(x_{i+1}(t\,;t_i, x(t_i+0\,;t_0,x_0))), \qquad (11)\\ & f_{i+1}(t, x_{i+1}(t\,;t_i, x(t_i+0\,;t_0,x_0)))\rangle|\\ \geq& C_{\langle grad\phi_{i+1}, f_{i+1}\rangle} \qquad (12)\\ =& \text{ const} > 0. \qquad (13)\end{aligned}$$

By (9) and (10), it follows that there exists a point $t_{i+1} > t_i$ such that

$$\varphi_{i+1}(t_{i+1}) = 0 \quad \Leftrightarrow \quad \phi_{i+1}(x_{i+1}(t_{i+1}\,; t_i, x(t_i + 0\,; t_0, x_0))) = 0.$$

The last equality implies that the trajectory $\gamma_{i+1}(t_i, x(t_i + 0\,; t_0, x_0))$ meets the hypersurface Φ_{i+1} at the moment t_{i+1}. Since

$$\gamma(t_0, x_0) \equiv \gamma_{i+1}(t_i, x(t_i + 0\,; t_0, x_0)) \text{ for } t_i < t \le t_{i+1},$$

we establish that the trajectory $\gamma(t_0, x_0)$ of problem (1), (2), (3), (4) also meets hypersurface Φ_{i+1}.

The proof follows by induction.

The theorem is proved. □

Theorem 3. *Let conditions H1–H7 be fulfilled.*
Then the following estimates are valid

$$t_{i+1} - t_i \le \frac{C_{\phi_{i+1}}}{C_{\langle grad\phi_{i+1}, f_{i+1}\rangle}}\ ,\ i = 1, 2, \ldots.$$

Proof. Let i be arbitrary natural number. We consider function φ_{i+1}, defined in interval $[t_i, t_{i+1}]$ using equality (8). More precisely, we have

$$\begin{aligned}\varphi_{i+1}(t) =& \phi_{i+1}(x_{i+1}(t\,; t_i, x(t_i + 0\,; t_0, x_0))) \\ =& \begin{cases} \phi_{i+1}(x(t_i + 0\,; t_0, x_0)), & t = t_i; \\ \phi_{i+1}(x(t\,; t_0, x_0)), & t_i < t \le t_{i+1}. \end{cases}\end{aligned}$$

Then using condition H7, we find

$$\begin{aligned} C_{\phi_{i+1}} \ge & |\phi_{i+1}((Id + I_i)(x(t_i; t_0, x_0)))| \\ =& |\phi_{i+1}(x(t_i + 0; t_0, x_0))| \\ =& |\phi_{i+1}(x(t_{i+1}; t_0, x_0)) - \phi_{i+1}(x(t_i + 0; t_0, x_0))| \\ =& |\varphi_{i+1}(t_{i+1}) - \varphi_{i+1}(t_i)| \\ =& \left|\frac{d}{dt}\varphi_{i+1}(t^*)\right|(t_{i+1} - t_i) \\ =& \left|\frac{\partial}{\partial x^1}\phi_{i+1}(x(t^*; t_0, x_0))\,\frac{d}{dt}x^1(t^*; t_0, x_0)\right. \\ & + \frac{\partial}{\partial x^2}\phi_{i+1}(x(t^*; t_0, x_0))\,\frac{d}{dt}x^2(t^*; t_0, x_0) + \cdots \\ & \left. + \frac{\partial}{\partial x^n}\phi_{i+1}(x(t^*; t_0, x_0))\,\frac{d}{dt}x^n(t^*; t_0, x_0)\right|(t_{i+1} - t_i) \end{aligned}$$

$$
\begin{aligned}
&= \bigg| \frac{\partial}{\partial x^1}\phi_{i+1}\left(x\left(t^*;t_0,x_0\right)\right) f^1_{i+1}\left(t^*,x\left(t^*;t_0,x_0\right)\right) \\
&\quad + \frac{\partial}{\partial x^2}\phi_{i+1}\left(x\left(t^*;t_0,x_0\right)\right) f^2_{i+1}\left(t^*,x\left(t^*;t_0,x_0\right)\right) + \cdots \\
&\quad + \frac{\partial}{\partial x^n}\phi_{i+1}\left(x\left(t^*;t_0,x_0\right)\right) f^n_{i+1}\left(t^*,x\left(t^*;t_0,x_0\right)\right) \bigg| \left(t_{i+1}-t_i\right) \\
&= \left|\left\langle grad\phi_{i+1}\left(x\left(t^*;t_0,x_0\right)\right), f_{i+1}\left(t^*,x\left(t^*;t_0,x_0\right)\right)\right\rangle\right| \left(t_{i+1}-t_i\right) \\
&\geq C_{\langle grad\phi_{i+1},f_{i+1}\rangle}\left(t_{i+1}-t_i\right),
\end{aligned}
$$

where point t^* satisfies the inequalities $t_i < t^* < t_{i+1}$.

The theorem is proved. □

Theorem 4. *Let conditions H1–H8 be fulfilled.*

Then the solution of system (1), (2), (3) dies due to the impulsive effects.

Proof. The next equality is valid

$$
\begin{aligned}
J\left(t_0,x_0,f\right) &= \left[t_0,t_1\right] \cup \left(t_1,t_2\right] \cup \left(t_2,t_3\right] \cup \ldots \\
&= \left[t_0,t^0\right),
\end{aligned}
$$

where

$$
\begin{aligned}
t^0 = \lim_{i\to\infty} t_i &= t_1 + \lim_{i\to\infty}\left(\left(t_2-t_1\right)+\left(t_3-t_2\right)+\ldots+\left(t_i-t_{i-1}\right)\right) \\
&= t_1 + \sum\nolimits_{i=1}^{\infty}\left(t_{i+1}-t_i\right) \\
&= t_1 + \sum\nolimits_{i=1}^{\infty}\frac{C_{\phi_{i+1}}}{C_{\langle grad\phi_{i+1},f_{i+1}\rangle}} \\
&< \infty.
\end{aligned}
$$

The theorem is proved. □

Remark 2. From condition H6, it follows that the solution of problem (1), (2), (3), (4) is probably to die due to the impulsive effects only in the case in which they are innumerable.

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