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THE STUDY OF A CLASS OF THE BROWNIAN DERIVATIVE SYSTEM

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Abstract: In this paper, we study the Brownian Derivative System: A class of Wiener processes and a class of Langevin random system using wavelet analysis, we obtain some properties and wavelet express.

Key Words: stochastic processes, Wiener processes, wavelet, Langevin random system

1. Introduction

The stochastic system is very important in many aspects. Wiener difference processes is a sort of important stochastic processes. Langevin is a class of useful stochastic processes in theory and practices, its study is very value.

We will take wavelet and use them in a series expansion of signal or function. Wavelet has its energy concentrated in time to give a tool for the analysis of transient, nonstationary, or time-varying phenomena. It still has the oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. We take wavelet and use them in a series expansion of signals or functions much the same way a Fourier series the wave or sinusoid to represent a signal or function. In order

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to use the idea of multiresolution, we will start by defining the scaling function and then define the wavelet in terms of it.

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision, One must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data. Wavelets have contributed to this already intensely developed and rapidly advancing field.

Wavelet analysis consists of a versatile collection of tools for the analysis and manipulation of signals such as sound and images as well as more general digital data sets, such as speech, electrocardiograms, images. Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering. The basic idea is always to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks themselves come in different sizes, and are suitable for describing features with a resolution commensurate with their size.

There are two important aspects to wavelets, which we shall call "mathematical" and "algorithmical". Numerical algorithms using wavelet bases are similar to other transform methods in that vectors and operators are expanded into a basis and the computations take place in the new system of coordinates. As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain, wavelet based algorithms exhibit a number of new and important properties. Recently some persons have studied wavelet problems of stochastic process or stochastic system.

2. Basic Definitions and Properties of Wiener Difference Processes

Definition 1. Let$\{W(t), t > 0\}$ is σ^2-Wiener processes, $a > 0$. Let

$$x(t) = w(a + t) - w(t) \tag{1}$$

We call $x(t)$ is Wiener difference processes. We have: $E(x(t)) = 0$,

$$\begin{aligned} E(x(s)x(t)) =& E(w(a + s)w(a + t)) - w(a + s)w(t) - w(a + t)w(s) + w(s)w(t)) \\ =& [min(a + s, a + t) - min(a + s, t) - min(a + t, s) + min(s, t)]\sigma^2 \end{aligned}$$

$$=\begin{cases} 0, & \text{if} a \leq |t-s|, \\ (a-|t-s|)\sigma^2, & \text{if} |t-s| < a. \end{cases} \tag{2}$$

If $t > s$, then we have

$$E(x(s)x(t)) = \begin{cases} 0, & \text{if} a \leq t-s, \\ (a-t+s)\sigma^2, & \text{if} t-s < a. \end{cases} \tag{3}$$

Definition 2. Let$\{x(t), t \in R\}$is stochastic processes, $E(x(t))^2 < +\infty$, then

$$w(s,x) = \frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt \tag{4}$$

is wavelet change of x(t), where ψ is mather wavelet.
Haarwavelet is

$$\psi(x) = \begin{cases} 1, & \text{if} 0 \leq x < \frac{1}{2}, \\ -1, & \text{if} \frac{1}{2} \leq x < 1, \\ 0, & \text{if} else. \end{cases} \tag{5}$$

Use(4), we have

$$w(s, x+\tau) = \frac{1}{s}\int_R x(t)\psi(\frac{x+\tau-t}{s})dt \tag{6}$$

then

$$\begin{aligned} R(\tau) &= E(w(s,x)w(s,x+\tau)) \\ * &= E(\frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt\frac{1}{s}\int_R x(t)\psi(\frac{x+\tau-t}{s})dt_1) \\ * &= \frac{1}{s^2}\int\int E(x(t)x(t_1))\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t}{s})dtdt_1 \end{aligned} \tag{7}$$

Use (5), we have

$$\psi(\frac{x-t}{s}) = \begin{cases} 1, & \text{if} x - \frac{s}{2} \leq t < x, \\ -1, & \text{if} x - s \leq t < x - \frac{s}{2}. \end{cases}$$

$$\psi(\frac{x+\tau-t}{s}) = \begin{cases} 1, & \text{if} x+\tau-\frac{s}{2} \leq t < x+\tau, \\ -1, & \text{if} x+\tau-s \leq t < x+\tau-\frac{s}{2}. \end{cases}$$

then

$$R(t) = \frac{1}{s^2}\int\int_{(u-t)>a}(a-t-u)\sigma^2\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t}{s})dtdu$$

$$
\begin{aligned}
* &= \sigma^2 \div (s^2(\int\limits_{x-\frac{s}{2}}^{x} (a-t+u)dt \int\limits_{t+a}^{x+\tau} du - \int\limits_{x-\frac{s}{2}}^{x} (a-t+u)dt \int\limits_{t+a}^{x+\tau-\frac{s}{2}})du \\
* &\quad - \int\limits_{x-s}^{x-\frac{s}{2}} (a-t+u)dt \int\limits_{t+a}^{x+\tau} du + \int\limits_{x-s}^{x-\frac{s}{2}} (a-t+u)dt \int\limits_{t+a}^{x+\tau} du)) \\
* &= I_1 + I_2 + I_3 + I_4
\end{aligned}
$$

where

$$
\begin{aligned}
I_1 &= \frac{1}{s^2}(\int\limits_{\frac{s}{2}}^{x} dt \int\limits_{t+a}^{x+\tau} (a-t+u)du) \\
&= \frac{1}{s^2} \int\limits_{x-\frac{s}{2}}^{x} ((a-t)(x+\tau-t-a) + \frac{1}{2}(x+\tau)^2 - \frac{1}{2}(t-a)^2)dt
\end{aligned}
$$

$$
\begin{aligned}
I_2 =& -\frac{1}{s^2} \int\limits_{x-\frac{s}{2}}^{x} (a-t+u)dt \int\limits_{t+a}^{x+\tau-\frac{s}{2}} du \\
=& -\frac{1}{s^2} \int\limits_{x-\frac{s}{2}}^{x} ((a-t)(x+\tau-\frac{s}{2}) + \frac{1}{2}(x+\tau-\frac{s}{2})^2 - (a-t)(a+t) \\
& - \frac{1}{2}(t+a)^2)dt
\end{aligned}
$$

$$
\begin{aligned}
I_3 &= -\frac{1}{s^2} \int\limits_{x-s}^{x-\frac{s}{2}} dt \int\limits_{t+a}^{x+\tau} (a-t+u)du \\
&= -\frac{1}{s^2} \int\limits_{x-s}^{x-\frac{s}{2}} ((a-t)(x+\tau) + \frac{1}{2}(x+\tau)^2 - (a-t)(a+t) - \frac{1}{2}(t+a)^2)dt
\end{aligned}
$$

$$\begin{aligned} I_4 &= \frac{1}{s^2}\int\limits_{x-s}^{x-\frac{s}{2}} dt \int\limits_{t+a}^{x+\tau-\frac{s}{2}} (a-t+u)du \\ &= \frac{1}{s^2}\int\limits_{x-s}^{x-\frac{s}{2}} ((a-t)(x+\tau-\frac{s}{2})+\frac{1}{2}(x+\tau-\frac{s}{2})-(a-t)(a+t) \\ &\quad -\frac{1}{2}(t+a)^2)dt \end{aligned}$$

then

$$I_1+I_2=\frac{1}{s^2}\int\limits_{x-\frac{s}{2}}^{x} ((a-t)\frac{s}{2}+s(x+\tau)-\frac{s^2}{8})dt$$

$$I_3+I_4=\frac{1}{s^2}\int\limits_{x-s}^{x-\frac{s}{2}} (-\frac{s}{2}(a-t)-s(x+\tau)+\frac{s^2}{8})dt$$

we have

$$\begin{aligned} R(\tau) &= \frac{1}{s^2}\int\limits_{x-s}^{x} (-\frac{s}{2}(a-t)-s(x+\tau)+\frac{s^2}{8})dt \\ &= \frac{1}{s}\int\limits_{x-s}^{x} (-\frac{1}{2}(a-t)-(x+\tau)+\frac{s}{8})dt \\ &= \frac{1}{s}\int\limits_{x-s}^{x} (\frac{1}{2}t-\frac{1}{2}a-x-\tau+\frac{s}{8})dt \\ &= -8(4a+4x+s+8\tau) \end{aligned}$$

then, we have

$$R'(\tau)=-64, R''(\tau)=0$$

Then the zero density degree of $W(s,x)$ is 0.
Let $\varphi(t)=\sqrt{2}\sum_k \varphi(2x-k)$
then, wavelet express of $x(t)$

$$x(t)=2^{-\frac{J}{2}}\sum_{n\in Z} C_n^J\varphi(2^{-J}t-n)+\sum_{j\leq J}2^{-\frac{j}{2}}\sum_{n\in Z} d_n^j\psi(2^{-j}t-n)$$

where

$$C_n^J = \int\limits_R x(t)\varphi(2^{-J}t - n)dt$$

$$d_n^j = \int\limits_R x(t)\psi(2^{-j}t - n)dt$$

then, for d_n^j, we have

$$\begin{aligned} E(d_n^j d_m^k) &= E\int\limits_R x(t)\psi(2^{-j}t - n)dt \int\limits_R x(s)\psi(2^{-k}s - m)ds \\ &= \iint\limits_{R^2} E(x(t)x(s))\psi(2^{-j}t - n)\psi(2^{-k}s - m)dtds \\ &= \int \int\limits_{|t-s|>a} (a - |t-s|\sigma^2\psi(2^{-j}t - n)\psi(2^{-k}s - m))dtds \end{aligned}$$

3. The Study the Langevin System

Definition 3. Langevin system is stochastic system as follows

$$dX(t) = -kX(t)dt + \frac{1}{m}cW(t) \tag{8}$$

Where k, m are contents. equation(8)changes to

$$\triangle x = -kX(t)\triangle t + \frac{W(t+\triangle t) - W(t)}{m} + o(\triangle t)$$

Where W(t) is Wiener processes. let $m = 1$, then

$$E(W(t+\triangle t) - W(t)) = 0$$

$$E((W(t+\triangle t) - W(t))^2) = 2D\triangle t$$

D is content, we have

$$E(\triangle x|X(t) = y) = -ky\triangle t + o(\triangle t)$$

$$E((\triangle x)^2|X(t) = y) = k^2y^2(\triangle t)^2 + 2D\triangle t + o(\triangle t)$$

then

$$lim\frac{1}{\triangle t}E(\triangle x|X(t)=y)=-ky,\triangle t\to 0.$$

$$lim\frac{1}{\triangle t}E((\triangle x)^2|X(t)=y)=2D,\triangle t\to 0.$$

We have $EX(t)=0$

$$E(X(s)X(t))=\frac{D}{k}e^{-k(s-t)}$$

Use equation(5), have

$$\begin{aligned}\psi(\frac{u-t}{s}) &= \begin{cases}1, & \text{if} 0<\frac{u-t}{s}\leq\frac{1}{2},\\ 0, & \text{if}\frac{1}{2}<\frac{u-t}{s}\leq t,\\ 0, & \text{if} else.\end{cases}\\ &= \begin{cases}1, & \text{if} u-\frac{s}{2}<t\leq u,\\ -1, & \text{if} u-s<t\leq u-\frac{s}{2},\\ 0, & \text{if} else.\end{cases}\end{aligned}$$

then

$$\begin{aligned}WX(s,u) &= \frac{1}{s}\int\limits_{R}X(t)\psi(\frac{u-t}{s})dt\\ &= \frac{1}{s}\int\limits_{u-\frac{s}{2}}^{u}X(t)dt-\frac{1}{s}\int\limits_{u-s}^{u-\frac{s}{2}}X(t)dt\end{aligned}$$

have

$$\begin{aligned}E(WX(s,x)) &= \frac{1}{s}\int\limits_{u-\frac{s}{2}}^{u}E(X(t))dt-\frac{1}{s}\int\limits_{u-s}^{u-\frac{s}{2}}E(X(t))dt\\ &= 0\end{aligned}$$

For WX(s, u), we have

$$\begin{aligned}R(t) &= E(WX(s,u)W(s,u+\tau))\\ &= E(\frac{1}{s}\int\limits_{R}X(t)\psi(\frac{u-t}{s})dt\frac{1}{s}\int\limits_{R}X(v)\psi(\frac{u+\tau-v}{s})dv)\\ &= \frac{1}{s^2}\int\int\limits_{R^2}E(X(t)X(v))\psi(\frac{u-t}{s})\psi(\frac{u+\tau-v}{s})dtdv\end{aligned}$$

because

$$E(X(t)X(v)) = \frac{D}{k}e^{-k(t-v)}$$

then

$$\begin{aligned} R(t) &= \frac{1}{s^2}\int\int\limits_{R^2}\frac{D}{k}e^{-k(t-v)}\psi(\frac{u-t}{s})\psi(\frac{u+\tau-v}{s})dtdv \\ &= \frac{1}{s^2}\frac{D}{k}\int\limits_{R} e^{-kt}\psi(\frac{u-t}{s})dt\int\limits_{R} e^{kv}\psi(\frac{u+\tau-v}{s})dv \end{aligned}$$

We have

$$\begin{aligned} R(t) &= \frac{1}{s^2}\frac{D}{k}(\int\limits_{u-\frac{s}{2}}^{u} e^{-kt}dt - \int\limits_{u-s}^{u-\frac{s}{2}} e^{-kt}dt)(\int\limits_{u+\tau-\frac{s}{2}}^{u+\tau} e^{kv}dv - \int\limits_{u+\tau-s}^{u+\tau-\frac{s}{2}} e^{kv}dv) \\ &= \frac{D}{ks^2}(-\frac{1}{k}(e^{-ku} - e^{-k(u-\frac{s}{2})}) + \frac{1}{k}(e^{-k(u-\frac{s}{2})} - e^{-k(u-s)})) \\ &\quad (\frac{1}{k}(e^{k}(u+\tau) - e^{k(u+\tau-\frac{s}{2})}) - \frac{1}{k}(e^{k(u+\tau-\frac{s}{2})} - e^{k(u+\tau-s)})) \\ &= \frac{D}{k^2s^2}(-6e^{k\tau} + 4e^{\frac{ks}{2}+k\tau} + 2e^{\frac{ks}{2}-k\tau} + 2e^{k\tau-\frac{ks}{2}} - e^{k\tau-ks} - e^{ks+k\tau}) \end{aligned}$$

We study the energy of the system, We have

$$R^{(1)}(\tau) = \frac{D}{ks^2}(-6e^{kt} + 4e^{\frac{ks}{2}+ks} - 2e^{\frac{ks}{2}-ks} + 2e^{k\tau-\frac{ks}{2}} - e^{k\tau-ks} - e^{ks+k\tau})$$

$$R^{(2)}(\tau) = \frac{D}{s^2}(-6e^{k\tau} + 4e^{\frac{ks}{2}+kv} + 2e^{\frac{ks}{2}-k\tau} + 2e^{k\tau-\frac{ks}{2}} - e^{k\tau-ks} - e^{ks+k\tau})$$

$$R^{(3)}(\tau) = \frac{kD}{s^2}(-6e^{k\tau} + 4e^{\frac{ks}{2}+k\tau} - 2e^{\frac{ks}{2}-k\tau} + 2e^{k\tau-\frac{ks}{2}} - e^{k\tau-ks} - e^{ks+k\tau})$$

$$R^{(4)}(\tau) = \frac{kD}{s^2}(-6e^{k\tau} + 4e^{\frac{ks}{2}+k\tau} + 2e^{\frac{ks}{2}-k\tau} + 2e^{k\tau-\frac{ks}{2}} - e^{k\tau-ks} - e^{ks+k\tau})$$

then

$$R(0) = \frac{D}{(ks)^2}(-6 + 6e^{\frac{ks}{2}} + 2e-\frac{ks}{2} - e^{-ks} - e^{ks})$$

$$R^{(2)}(0) = \frac{D}{s^2}(-6 + 6e^{\frac{ks}{2}} + 2e-\frac{ks}{2} - e^{-ks} - e^{ks})$$

$$R^{(4)}(0) = \frac{D}{s^2}(-6 + 6e^{-\frac{ks}{2}} + 2e-\frac{ks}{2} - e^{-ks} - e^{ks})$$

The zero density degree of WX(s, u)is

$$W_0 = (|\frac{R^{(2)}(0)}{\pi^2 R(0)}|)^{\frac{1}{2}}$$

The average density degree of WX(s, u) is

$$W_1 = (|\frac{R^{(4)}(0)}{\pi^2 R^{(2)}(0)}|)^{\frac{1}{2}}$$

Then we have:

$$W_0 = \frac{k}{\pi}, W_1 = \frac{1}{\pi}.$$

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